

Modelling the Golf Downswing as a Driven Double Pendulum: A Lagrangian Analysis of Wrist Release Timing and Clubhead Speed

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ABSTRACT

The distance of a golf drive is determined primarily by the speed of the clubhead at the moment of impact. In this paper, the downswing of a driver shot is modelled as a driven double pendulum, consisting of a rigid arm link rotating about a fixed hub and a rigid club link connected at the wrists. The links move in a single plane under a constant hub torque. The equations of motion of the system are derived using Lagrangian mechanics and solved numerically using the fourth-order Runge–Kutta (RK4) method, as no closed-form solution exists. The model is used to investigate how the timing of the wrist release affects the clubhead speed at impact. For a baseline swing with a hub torque of 100 N m and an initial wrist-cock angle of 90°, the model predicts a natural release after the arm has swung through 46.1° and a clubhead speed at impact of 45.0 m s⁻¹ (100.7 mph), consistent with measured amateur and professional driver speeds. Delaying the release until the arm has swung through 118° increases the clubhead speed by 4.4%, while increasing the initial wrist-cock angle and the hub torque produce further, sub-proportional gains. A sensitivity analysis shows that the delayed-release gain remains between 3.0% and 6.0% when each model parameter is varied by a generous margin. These results agree closely with published double pendulum studies of the golf swing and provide a physical explanation for the common coaching instruction to “hold the lag” during the downswing.

INTRODUCTION

Golf is a sport that uses different clubs to hit a ball into a hole in the least number of shots possible. One of the main objectives in golf is to maximise the distance of the tee shot, as it leaves the golfer closer to the hole for their second shot. For a driver shot, this distance is determined almost entirely by the conditions of the clubhead at the moment of impact. The single most important of these conditions is clubhead speed, defined as the linear speed of the centre of the club face just before it contacts the ball (Trackman, n.d.). According to launch monitor data, each additional 1 mph of clubhead speed is worth up to 3 yards of extra distance (Trackman, n.d.), which is why nearly all modern driver coaching is focused on generating speed.

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As an avid golfer myself, one instruction I have been given repeatedly is to “hold the lag”: to keep my wrists cocked for as long as possible during the downswing and to avoid “casting” the club early from the top of the swing. When I asked why this works, the answers I received were mostly based on feel rather than physics, which made me wonder whether the advice could actually be justified using mechanics. After doing some research, I found that the golf downswing is actually a well-studied problem in physics: since the 1960s, researchers have modelled the arms and club as a double pendulum. A double pendulum is a system of two rigid links connected by hinges, and this simple model has been remarkably successful at reproducing the swings of professional golfers (Cochran & Stobbs, 1968; Jorgensen, 1970).

However, the equations of motion of a driven double pendulum are coupled and nonlinear, meaning they cannot be solved with the standard kinematics and dynamics methods used at high school level. This motivated me to derive the equations using Lagrangian mechanics, an energy-based reformulation of Newton’s laws that is particularly suited to linked systems, and to solve them numerically. My aim is to answer the following research question:

“How does the timing of the wrist release during the downswing affect the clubhead speed of a driver shot at impact, and what release strategy maximises this speed according to a double pendulum model of the golf swing?”

For this research question, the analysis ends at the moment of impact. The subsequent flight of the ball, which depends on aerodynamic effects such as drag and lift, is a separate problem and is beyond the scope of this paper.

BACKGROUND

The Double Pendulum Model of the Golf Swing

The idea of treating the golf swing as a double pendulum was popularised by Cochran and Stobbs in their book *Search for the Perfect Swing* (Cochran & Stobbs, 1968). Around the same time, Williams analysed stroboscopic photographs of the legendary golfer Bobby Jones and showed that his downswing was consistent with a two-link model driven by an approximately constant torque (Williams, 1967). Jorgensen then formalised the model, derived its equations of motion using Lagrangian mechanics, and demonstrated that it could match the measured positions of a real golfer’s arm and club throughout the downswing (Jorgensen, 1970; Jorgensen, 1999). Later authors extended the model to study ball position and release timing (Pickering & Vickers, 1999), active wrist torque (Sprigings & Neal, 2000), the “parametric acceleration” of the clubhead (Miura, 2001) and swing efficiency (White, 2006), while reviews of the field are given by Penner and Betzler et al. (Penner, 2003; Nils Betzler et al., 2008).

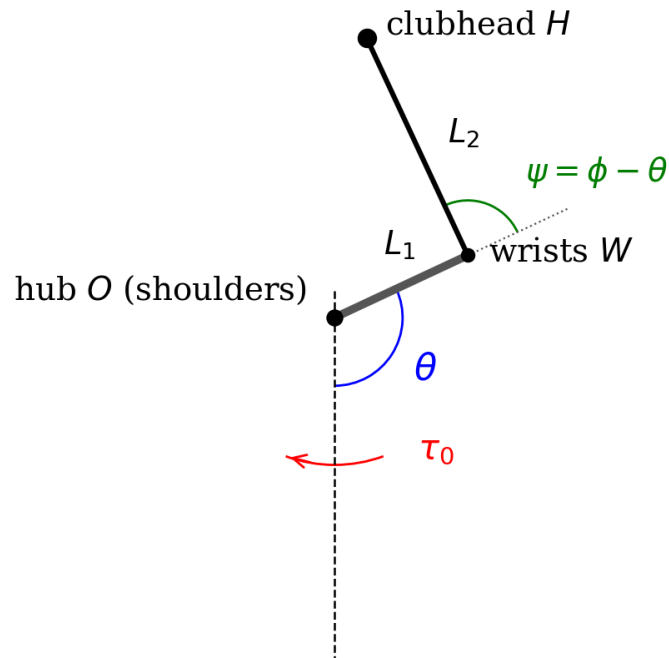


Figure 1: Diagram of the double pendulum model of the downswing, showing the hub O , wrists W , clubhead H , the arm angle θ , the wrist-cock angle $\psi = \phi - \theta$ and the hub torque τ_0 . Both θ and the club angle ϕ are measured anticlockwise from the downward vertical.

Some terminology used in the following sections (see **Figure 1** for illustration):

- **Hub:** the fixed pivot point between the shoulders about which the arms rotate.
- **Downswing:** the part of the swing from the top of the backswing to impact, lasting roughly 0.2–0.3 s (Penner, 2003).
- **Wrist-cock angle:** the angle between the line of the arms and the shaft of the club. A fully “uncocked” position, where the arms and club form a straight line, corresponds to a wrist-cock angle of 0° .
- **Lag:** the state in which the club trails behind the arms during the downswing, i.e. the wrist-cock angle remains large.
- **Release:** the moment in the downswing when the wrists stop holding the club back and allow it to swing out freely.

- **Natural release:** the specific moment when the torque required from the wrists to maintain the wrist-cock falls to zero, so that the club would begin to swing out on its own even with completely passive wrists.
- **Late hit (delayed release):** deliberately holding the wrist-cock beyond the natural release point.
- **Casting:** the opposite fault, where the wrist-cock is thrown away early in the downswing.
- **Hub torque:** the torque applied about the hub by the golfer's body (shoulders, torso and legs), which drives the whole system around.

A First Attempt Using Newton's Laws

My first attempt at deriving the equations of motion used Newton's laws directly, as this is the standard method at high school level. This approach requires separating the system into its two links and drawing a free body diagram of each one, shown in **Figure 2**.

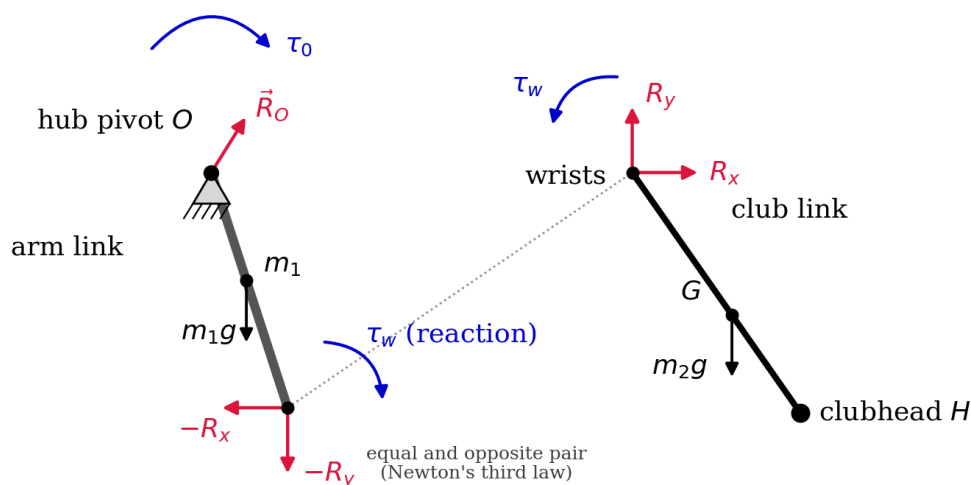


Figure 2: Free body diagrams of the two links separated at the wrist joint. The wrist reaction force, with components R_x and R_y , and the wrist torque τ_w act on the club and on the arm as equal and opposite pairs by Newton's third law. The hub reaction $\vec{R}_O$ acts on the arm at the fixed pivot O and is eliminated by taking torques about O .

The forces and torques acting on the links are:

- **Weights:** the arm's effective weight m_1g and the club's weight m_2g , where m_1 is the effective mass of the arms, m_2 is the mass of the club, g is the gravitational acceleration and G is the club's centre of mass.
- **Applied torques:** the hub torque τ_0 acting on the arm, and the wrist torque τ_w exerted by the arms on the club, whose reaction acts back on the arm.
- **Wrist reaction force:** the two links also push and pull on each other at the wrist joint with a contact force of unknown magnitude and direction, written in components as R_x and R_y . By Newton's third law, this force acts on the club and on the arm as an equal and opposite pair.
- **Hub reaction force:** the hub pivot also exerts a force $\vec{R}_O$ on the arm. Since the arm rotates about the fixed point O , this force can be eliminated in the usual way by taking torques about O .

However, the same method cannot be used on the club. This is because the club's pivot, the wrists, is itself accelerating as the arm swings, and taking torques about an accelerating point is not valid without additional correction terms. Therefore, the club must be treated with the full set of rigid body equations: Newton's second law applied to its centre of mass in each direction, plus the torque equation about the centre of mass. Writing (x_G, y_G) for the coordinates of the club's centre of mass, ϕ for the angle of the shaft measured anticlockwise from the downward vertical, r_2 for the distance from the wrists to the centre of mass and I_2 for the club's moment of inertia about it, these equations are:

$$m_2 \ddot{x}_G = R_x$$

$$m_2 \ddot{y}_G = R_y - m_2 g$$

$$I_2 \ddot{\phi} = \tau_w - r_2 (R_x \cos \phi + R_y \sin \phi)$$

These equations contain four unknowns: the two angular accelerations of the arm and of the club, which are the quantities we actually want to find, and the two reaction components R_x and R_y , which are not relevant to the research question but cannot simply be ignored. Furthermore, before any elimination can begin, x_G and y_G must be written in terms of the two angles and differentiated twice using the product and chain rules, which generates a large number of terms. This method does work, as combining the equations above to eliminate R_x and R_y eventually produces the same club equation derived later using Lagrangian mechanics. However, the algebra is long and prone to errors, and most of the working is spent dealing with the internal forces, which are not relevant to the research question.

Lagrangian mechanics avoids this problem. The wrist reaction is known as a constraint force, and because the action and reaction forces act at the same point, the pair does zero total work as the joint moves. Since the Lagrangian method is built entirely from energies, forces that do no work will never appear in its equations. Therefore, I decided to derive the equations of motion using Lagrangian mechanics instead, which is explained in the next section. We will also see later that the wrist torque τ_w can be eliminated when needed by simply adding the two equations of motion together.

Lagrangian Mechanics

Lagrangian mechanics is a reformulation of Newton's laws that works with energies, which are scalars, instead of forces, which are vectors. This means the constraint forces identified above, which do no work, automatically disappear from the equations.

The method works as follows. First, the configuration of the system is described by a set of generalised coordinates: the smallest set of variables needed to specify the position of every part of the system. For the double pendulum, two angles are sufficient. Next, the total kinetic energy T and total potential energy V of the system are written in terms of these coordinates and their time derivatives, and combined into a single function called the Lagrangian:

$$L = T - V$$

The equation of motion for each generalised coordinate q is then given by the Euler–Lagrange equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q$$

where Q is the generalised force (for an angular coordinate, a torque) applied externally to that coordinate. For a single particle moving in one dimension with $L = \frac{1}{2}mv^2 - V(x)$, this equation reduces exactly to Newton's second law $ma = F$, so the Euler–Lagrange equation can be thought of as a generalisation of $F = ma$ that works in any coordinate system. A key advantage for this investigation is that the internal forces at the wrist joint never appear in the equations: only the physically meaningful applied torques do.

MODEL

Assumptions and Justification

Assumptions (used to simplify the model):

- **The swing is planar.** The arms and club move in a single fixed plane. Real swing planes are inclined at roughly 45°–60° to the horizontal, but treating the motion as planar is the standard simplification made in all classical double pendulum studies (Jorgensen, 1970; Penner, 2003). For simplicity, the plane is taken to be vertical, so the full value of g acts within it. Since gravity

supplies only around 12% of the final kinetic energy (see the Results section), this is expected to overestimate the clubhead speed by no more than 1–2% and should not change which release timing is best.

- **The two links are rigid.** The left arm is kept straight in a good golf swing (for a right-handed golfer), so the two arms and shoulders are treated as a single rigid link, and the deflection of the club shaft is neglected. Milne and Davis found that the shaft plays only a minor role in generating clubhead speed (Milne & Davis, 1992), so this assumption mainly affects the delivered loft rather than the speeds predicted here.
- **The hub is fixed.** The pivot point between the shoulders does not translate during the downswing. Real golfers shift their hub towards the target during the downswing, and Jorgensen showed that this shift adds an energy contribution comparable to gravity's (Jorgensen, 1999). The fixed hub is therefore expected to underestimate the absolute clubhead speed by roughly 5%, although it should affect all release timings similarly, so the predicted optimum release angle should not change much.
- **The arms are modelled as an effective point mass.** The arm mass is treated as a single point mass m_1 located at the wrist radius L_1 . This is the simplification used by White, who notes that the exact mass distribution of the arms is difficult to measure and that an effective value fitted from typical swings is sufficient for this type of model (White, 2006). This means the absolute speeds are less certain than the trends, which is tested directly in the sensitivity analysis in the Results section.
- **The club is a rigid body.** The club has mass m_2 , centre of mass a distance r_2 from the wrists, and moment of inertia I_2 about its own centre of mass. Unlike the arms, the club's rotation about its centre of mass contributes significantly to the dynamics, so it is not reduced to a point mass.
- **The hub torque τ_0 is constant.** The torque applied by the body is treated as constant throughout the downswing. Williams showed that a constant torque reproduces the swing of Bobby Jones well (Williams, 1967), and both Jorgensen and White adopt the same assumption (Jorgensen, 1999; White, 2006). The main effect of this assumption is on the timing of the swing rather than on the speeds, as a real golfer's torque builds up through the transition; this is revisited in the Discussion section.
- **The wrists act in two stages.** During the first stage of the downswing, the wrist-cock angle is held fixed at its initial value ψ_0 . At the release, the wrists become a completely free, frictionless hinge that applies zero torque. This two-stage structure follows Jorgensen and Pickering & Vickers (Jorgensen, 1999; Pickering & Vickers, 1999). Since the wrists apply zero torque after the release, the model gives a lower bound on the speed: Sprigings and Neal showed that an

optimally timed active wrist torque can add up to 9% more clubhead speed (Sprigings & Neal, 2000).

- **Impact occurs when the club shaft passes through the vertical.** Simply put, impact occurs when the clubhead is directly below the wrists. The ball is assumed to be positioned at this point, which is slightly ahead of the hub in the direction of the swing, similar to the forward ball position golfers use for a driver shot.
- **Gravity is included; air resistance on the club is not.** The aerodynamic drag on the club during the ~0.3 s downswing is small compared to the applied torque, whereas gravity is known to contribute over 10% of the clubhead's final kinetic energy (Jorgensen, 1999), so it is kept in the model.

Coordinates and Setup

We first define the two generalised coordinates of the system (see **Figure 1**). Let θ be the angle of the arm link and ϕ be the angle of the club shaft, both measured anticlockwise from the downward vertical through the hub O . The wrist-cock angle is then $\psi = \phi - \theta$. The arm link has length L_1 from the hub to the wrists W , and the club extends a length L_2 from the wrists to the clubhead H .

At the top of the backswing, the system is at rest with the arm at $\theta(0) = \theta_0$ and the club cocked at $\psi(0) = \psi_0$, so $\phi(0) = \theta_0 + \psi_0$. It is also useful to define the swung angle $\delta = \theta_0 - \theta$, which is the angle the arm has rotated through since the top of the backswing. The release angle δ_r is the value of δ at the moment the wrists release, and the natural release angle δ_n is the value of δ at which the required wrist torque falls to zero. The golfer's body then applies the constant hub torque, which acts in the clockwise (negative) direction to drive the arm from θ_0 down towards $\theta = 0$; in the equations of motion this appears as a generalised torque $-\tau_0$ with $\tau_0 > 0$. The swing proceeds in the two stages described in the assumptions above: first with the wrist-cock held fixed, then, after the release, with the wrists acting as a free hinge. Impact occurs when $\phi = 0$.

DERIVATION OF THE EQUATIONS OF MOTION

Kinetic and Potential Energy

Taking the hub O as the origin, with x horizontal and y vertically upwards, the position of the wrists is:

$$x_W = L_1 \sin \theta, \quad y_W = -L_1 \cos \theta$$

and the position of the club's centre of mass G is:

$$x_G = L_1 \sin \theta + r_2 \sin \phi, \quad y_G = -L_1 \cos \theta - r_2 \cos \phi$$

Differentiating with respect to time and using the chain rule gives the velocity components, for example $\dot{x}_G = L_1 \dot{\theta} \cos \theta + r_2 \dot{\phi} \cos \phi$. The speed of G then follows from $v_G^2 = \dot{x}_G^2 + \dot{y}_G^2$, and after expanding and applying the identity $\cos \phi \cos \theta + \sin \phi \sin \theta = \cos(\phi - \theta)$:

$$v_G^2 = L_1^2 \dot{\theta}^2 + r_2^2 \dot{\phi}^2 + 2L_1 r_2 \dot{\theta} \dot{\phi} \cos(\phi - \theta)$$

The total kinetic energy is the sum of the kinetic energy of the arm mass, the translational kinetic energy of the club's centre of mass, and the rotational kinetic energy of the club about its centre of mass:

$$T = \frac{1}{2} m_1 L_1^2 \dot{\theta}^2 + \frac{1}{2} m_2 v_G^2 + \frac{1}{2} I_2 \dot{\phi}^2$$

Let $A = (m_1 + m_2)L_1^2$, $B = m_2 L_1 r_2$ and $C = I_2 + m_2 r_2^2$ to simplify the equations by combining constants into single parameters. Substituting v_G^2 and collecting terms gives:

$$T = \frac{1}{2} A \dot{\theta}^2 + \frac{1}{2} C \dot{\phi}^2 + B \dot{\theta} \dot{\phi} \cos(\phi - \theta)$$

The potential energy is found from the heights of the two masses:

$$V = -(m_1 + m_2)g L_1 \cos \theta - m_2 g r_2 \cos \phi$$

Therefore, the Lagrangian of the system is:

$$L = \frac{1}{2} A \dot{\theta}^2 + \frac{1}{2} C \dot{\phi}^2 + B \dot{\theta} \dot{\phi} \cos(\phi - \theta) + (m_1 + m_2)g L_1 \cos \theta + m_2 g r_2 \cos \phi$$

Applying the Euler–Lagrange Equations

Before applying the Euler–Lagrange equations, we need to identify the generalised torques. The hub torque acts on the arm in the negative direction, contributing $-\tau_0$ to Q_θ . The wrist torque τ_w is exerted by the arms on the club, contributing $+\tau_w$ to Q_ϕ ; by Newton's third law, the club exerts the reaction $-\tau_w$ back on the arms, contributing $-\tau_w$ to Q_θ . Therefore:

$$Q_\theta = -\tau_0 - \tau_w, \quad Q_\phi = +\tau_w$$

We now apply the Euler–Lagrange equation to the coordinate θ . The first step is to differentiate the Lagrangian with respect to $\dot{\theta}$:

$$\frac{\partial L}{\partial \dot{\theta}} = A\dot{\theta} + B\dot{\phi}\cos(\phi - \theta)$$

Differentiating this with respect to time (using the product and chain rules on the second term) gives:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) = A\ddot{\theta} + B\ddot{\phi}\cos(\phi - \theta) - B\dot{\phi}(\dot{\phi} - \dot{\theta})\sin(\phi - \theta)$$

Next, we differentiate the Lagrangian with respect to θ itself:

$$\frac{\partial L}{\partial \theta} = B\dot{\theta}\dot{\phi}\sin(\phi - \theta) - (m_1 + m_2)gL_1\sin\theta$$

Substituting these into the Euler–Lagrange equation and simplifying (the $B\dot{\theta}\dot{\phi}\sin(\phi - \theta)$ terms cancel out) gives the equation of motion for the arm:

$$A\ddot{\theta} + B\ddot{\phi}\cos(\phi - \theta) - B\dot{\phi}^2\sin(\phi - \theta) + (m_1 + m_2)gL_1\sin\theta = -\tau_0 - \tau_w$$

Following exactly the same procedure for the coordinate ϕ gives the equation of motion for the club:

$$C\ddot{\phi} + B\ddot{\theta}\cos(\phi - \theta) + B\dot{\theta}^2\sin(\phi - \theta) + m_2gr_2\sin\phi = \tau_w$$

These two equations are the coupled equations of motion of the driven double pendulum. During stage 2 of the downswing, the wrists are a free hinge, so we simply set $\tau_w = 0$ in both equations.

Stage 1: Holding the Wrist-Cock and the Natural Release

During stage 1, the wrist-cock is held fixed, so $\phi = \theta + \psi_0$ at all times, which means $\dot{\phi} = \dot{\theta}$ and $\ddot{\phi} = \ddot{\theta}$. Substituting these constraints into the two equations of motion and adding them together eliminates the internal wrist torque τ_w entirely (as it appears with opposite signs), leaving a single equation for a rigid body rotating about the hub:

$$I_{rig}\ddot{\theta} = -\tau_0 - (m_1 + m_2)gL_1\sin\theta - m_2gr_2\sin(\theta + \psi_0)$$

where $I_{rig} = A + C + 2B\cos\psi_0$ is the moment of inertia of the whole cocked system about the hub. Note that a larger wrist-cock angle ψ_0 makes $\cos\psi_0$ smaller, reducing I_{rig} . This means that cocking the wrists pulls the club's mass closer to the hub, making the system easier to accelerate.

We can also rearrange the club equation under the same constraints to find the wrist torque required to keep the club cocked:

$$\tau_w = (C + B \cos \psi_0) \ddot{\theta} + B \dot{\theta}^2 \sin \psi_0 + m_2 g r_2 \sin(\theta + \psi_0)$$

This equation contains the physics behind “lag”. Early in the downswing, $\dot{\theta}$ is small and $\ddot{\theta}$ is large and negative, so τ_w is negative: the wrists must actually drag the club along with the arms, because the club’s inertia makes it want to fall further behind (cock even more). As the swing speeds up, the centrifugal term $B \dot{\theta}^2 \sin \psi_0$ grows rapidly and τ_w becomes positive: the club now wants to swing outwards, and the wrists must hold it back to maintain the lag. The natural release is defined as the moment $\tau_w = 0$, when holding the cock stops requiring any effort and the club begins to release itself.

NUMERICAL METHOD

Parameters

The values of all constants used in the model are given in **Table 1** below.

Table 1: Parameter values used in the model and their sources.

Parameter	Symbol	Value	Source
Effective arm mass	m_1	3.0 kg	(White, 2006)
Hub-to-wrist length	L_1	0.67 m	(White, 2006)
Club mass	m_2	0.382 kg	(Nesbit, 2005)
Club length	L_2	1.092 m	(Nesbit, 2005)
Wrist to club centre of mass	r_2	0.661 m	(Nesbit, 2005)
Club moment of inertia (about CM)	I_2	0.071 kg m ²	(Nesbit, 2005)
Gravitational acceleration	g	9.81 m s ⁻²	(standard value)
Hub torque (baseline)	τ_0	100 N m	(White, 2006)
Backswing angle	θ_0	166°	(Jorgensen, 1999)
Initial wrist-cock angle (baseline)	ψ_0	90°	(White, 2006)

For context, fitted hub torques in the literature include 77.3 N m for Jorgensen’s professional golfer (Jorgensen, 1999) and 81–95 N m for a long-drive champion (Reyes & Mittendorf, 1999), so the baseline of 100 N m represents a strong but realistic swing. Substituting the values in **Table 1** gives the combined

constants $A = 1.518 \text{ kg m}^2$, $B = 0.169 \text{ kg m}^2$ and $C = 0.238 \text{ kg m}^2$, and a stage 1 moment of inertia of $I_{rig} = 1.756 \text{ kg m}^2$ for the baseline wrist-cock of 90° .

Solving the Equations with the Runge–Kutta Method

The two equations of motion contain products of the angular velocities and sines and cosines of the difference $\phi - \theta$, making them coupled and nonlinear. Unlike the equations of simple projectile motion or even a single pendulum with small angles, this system has no closed-form solution (an exact answer expressed as a finite combination of basic functions), so it must be integrated numerically.

To do this, the two second-order equations are first reduced to a system of four first-order equations by defining the state vector $u = (\theta, \dot{\theta}, \phi, \dot{\phi})$. At each instant, the two equations of motion form a pair of simultaneous linear equations in the two unknowns $\ddot{\theta}$ and $\ddot{\phi}$, which are solved algebraically, giving the derivative function $\dot{u} = f(u)$. The system is then advanced in small time steps h using the classical fourth-order Runge–Kutta (RK4) method (Press et al., 2007):

$$\begin{aligned} k_1 &= f(u_n) \\ k_2 &= f\left(u_n + \frac{h}{2}k_1\right) \\ k_3 &= f\left(u_n + \frac{h}{2}k_2\right) \\ k_4 &= f(u_n + hk_3) \\ u_{n+1} &= u_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \end{aligned}$$

The Runge–Kutta method was chosen over the simpler Euler method because its error per step scales with h^5 rather than h^2 , giving far higher accuracy for the same step size. A step size of $h = 1 \times 10^{-5} \text{ s}$ was used, which corresponds to around 34 000 steps across the $\sim 0.34 \text{ s}$ downswing.

The simulation proceeds in the two stages of the model. For reproducibility, the exact procedure is:

- **Initial conditions:** the system starts at rest at the top of the backswing, with $\theta(0) = \theta_0 = 166^\circ$, $\dot{\theta}(0) = 0$, $\phi(0) = \theta_0 + \psi_0$ and $\dot{\phi}(0) = 0$.
- **Stage 1 and the release:** the stage 1 rigid-body equation is integrated from rest until the chosen release condition is met. For the natural release, the release is triggered at the first time step where the required wrist torque τ_w changes sign from negative to positive. For a set release angle, the release is triggered at the first time step where the swung angle $\delta = \theta_0 - \theta$ reaches δ_r .

- **Stage 2:** the full coupled system is then integrated with $\tau_w = 0$. The handover is continuous: stage 2 starts from the stage 1 values of θ and $\dot{\theta}$, with $\phi = \theta + \psi_0$ and $\dot{\phi} = \dot{\theta}$.
- **Impact detection:** impact is taken as the first crossing of $\phi = 0$ from above, with linear interpolation between the two neighbouring time steps used to find the exact impact state. The clubhead velocity is then calculated from $\vec{v}_H = L_1 \dot{\theta} (\cos\theta, \sin\theta) + L_2 \dot{\phi} (\cos\phi, \sin\phi)$.
- **Parameter sweeps:** the release angle sweep uses δ_r from 20° to 158° in steps of 2° , and the wrist-cock sweep uses ψ_0 from 50° to 130° in steps of 5° .

Two checks were performed to verify the implementation, which was written in Python using the NumPy library (Harris et al., 2020), with all figures produced using Matplotlib (Hunter, 2007). The core simulation code is provided in the **Appendix**. First, halving the step size to $h = 5 \times 10^{-6}$ s changed the predicted impact speed by a relative amount of only 1.5×10^{-10} , confirming the solution has converged. Second, energy bookkeeping was tracked throughout stage 2: the increase in mechanical energy of the system agreed with the work done by the hub torque to within 0.01 J out of a total of 371 J (0.003%), confirming that the derived equations conserve energy correctly.

RESULTS

The Baseline Swing

The baseline swing uses the parameters of **Table 1** with the natural release condition. **Figure 3** shows the resulting motion as a stroboscopic diagram, with the positions of the arm and club drawn every 20 ms.

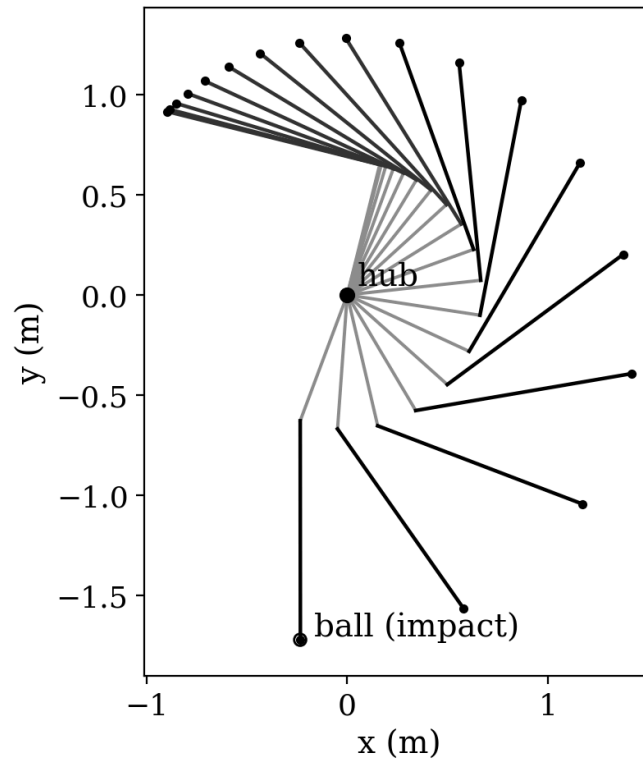


Figure 3: Stroboscopic diagram of the baseline downswing, with link positions drawn every 20 ms from the top of the backswing to impact. The club (black) visibly lags behind the arm (grey) for most of the downswing before whipping through in the final 100 ms.

The qualitative behaviour matches the swings of real golfers photographed by Jorgensen (Jorgensen, 1999). The club stays cocked and trails the arms deep into the downswing, then unwinds extremely rapidly just before impact. **Figure 4** plots the two angles and the clubhead speed against time, showing this behaviour quantitatively.

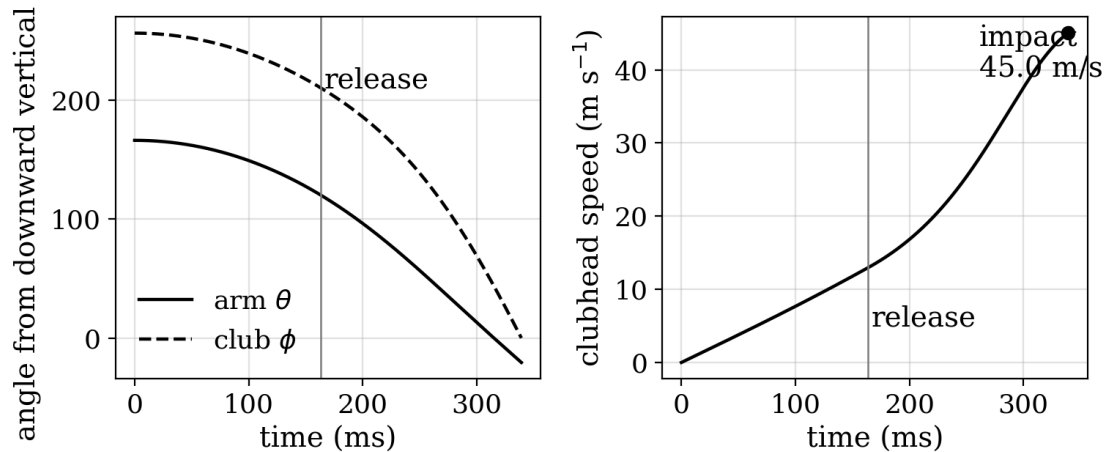


Figure 4: (Left) Arm angle θ and club angle ϕ against time for the baseline swing. (Right) Clubhead speed against time. The natural release occurs at 163.5 ms and impact at 339 ms, with the speed rising steeply only in the final phase of the downswing.

For the baseline swing, the natural release occurs after the arm has swung through $\delta_n = 46.1^\circ$, at $t = 163.5$ ms. Impact occurs at $t = 339$ ms with a clubhead speed of 45.0 ms^{-1} , equivalent to 100.7 mph. At the moment of impact, the club is rotating at $\dot{\phi} = 32.5 \text{ rad s}^{-1}$ while the arm has slowed to $\dot{\theta} = 14.9 \text{ rad s}^{-1}$, less than half the club's rate, and the club carries approximately 60% of the system's 371 J of kinetic energy. This shows that energy generated by the hub torque early in the downswing is stored in the rotating arm and club system and then transferred into the club during the release, creating the “whip” effect that makes the double pendulum model so effective at generating speed. Gravity contributes 45.5 J of this energy, or approximately 12%, which is very close to the 13% share Jorgensen calculated for his professional golfer (Jorgensen, 1999). It is also worth noting that at impact the arm has rotated 20.5° past the vertical while the shaft is vertical, meaning the ball is contacted around 0.23 m ahead of the hub in the direction of the swing. This is consistent with the forward ball position that golfers use for a driver.

The Natural Release

Figure 5 shows the wrist torque required to hold the cock, calculated from the holding torque equation, as the downswing progresses.

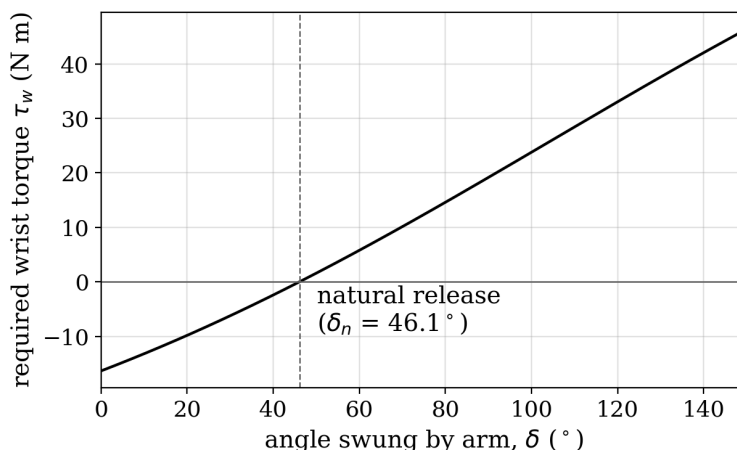


Figure 5: The wrist torque required to maintain a 90° wrist-cock, plotted against the angle δ swung by the arm since the top of the backswing. The torque starts at approximately -16 N m and crosses zero at

$$\delta_n = 46.1^\circ, \text{ the natural release point.}$$

The behaviour predicted in the derivation is clearly visible. For the first 46.1° of the downswing the required torque is negative: the wrists must pull the club along to stop it cocking even further. Beyond this point the required torque becomes positive and grows steeply, passing 45 N m by $\delta = 150^\circ$, which is why maintaining lag deep into the downswing requires real physical effort and is experienced by golfers as actively “holding on”.

To test the importance of the stage 1 constraint, the model was also run with completely passive wrists from the very top of the swing. In this case the wrist-cock does not disappear; instead, it grows, reaching a maximum of 161.5° , far beyond the range of motion of human wrists, and the resulting impact speed drops to 37.2 m s^{-1} . This confirms that some active control of the wrist angle is both physically necessary and beneficial, justifying the two-stage structure of the model.

Effect of Release Timing

We can now answer the main research question by prescribing the release angle δ_r (defined in the Coordinates and Setup sub-section) and sweeping it across its full range. The results are shown in **Figure 6**.

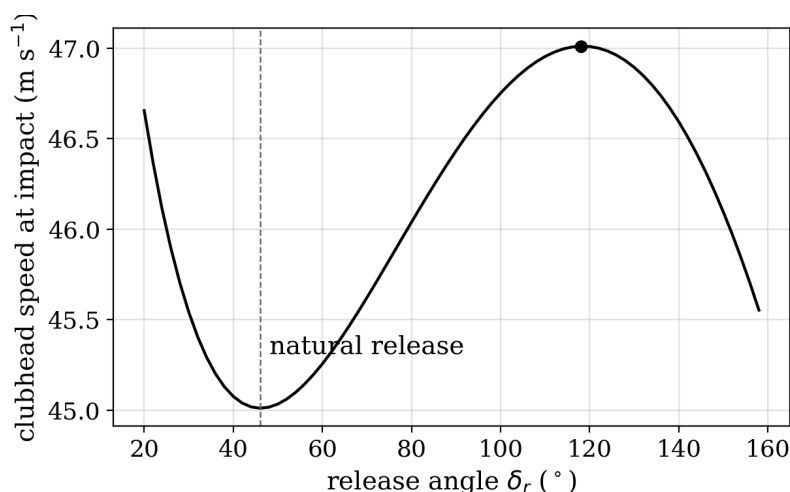


Figure 6: Clubhead speed at impact against release angle δ_r . The dashed line marks the natural release (46.1°). Delaying the release increases the impact speed up to a maximum of 47.0 m s^{-1} at $\delta_r = 118^\circ$, a gain of 4.4% over the natural release.

From **Figure 6**, three regions can be identified. First, releasing slightly earlier than the natural point makes almost no difference (within $+0.1\%$ at $\delta_r = 40^\circ$), because just before the natural release the free club barely opens on its own, so removing the constraint early hardly changes the motion. For very early releases the speed rises again (to 46.7 m s^{-1} at $\delta_r = 20^\circ$), because the freed club first cocks a few degrees beyond ψ_0 before swinging out, which acts like a slightly larger initial wrist-cock. However, this branch heads towards the fully passive case described above, where the over-cocking becomes extreme and the speed collapses, so it is not a practical strategy and the rest of this paper focuses on the delayed side. Second, holding the cock beyond the natural release steadily increases the impact speed: by $+2.3\%$ at $\delta_r = 80^\circ$, $+3.9\%$ at 100° , and a maximum of $+4.4\%$ (47.0 m s^{-1} , or 105.2 mph) at $\delta_r = 118^\circ$. Third, holding on even longer becomes counterproductive, with the speed falling back to $+3.5\%$ at $\delta_r = 140^\circ$, because the club does not have enough time left in the downswing to fully unwind.

However, the delayed release comes at a cost to the impact geometry. At the natural release, the club trails the arm by 20.5° at impact, but this lag angle grows to 32.8° at $\delta_r = 100^\circ$ and 50.6° at $\delta_r = 140^\circ$. In a real swing, reaching the ball with the club still lagging this far behind the arms would leave the club face open at impact, so the practically usable gain lies in the moderate delay region rather than at the mathematical maximum.

Effect of the Initial Wrist-Cock Angle

Figure 7 shows the effect of varying the initial wrist-cock angle ψ_0 while keeping the natural release condition.

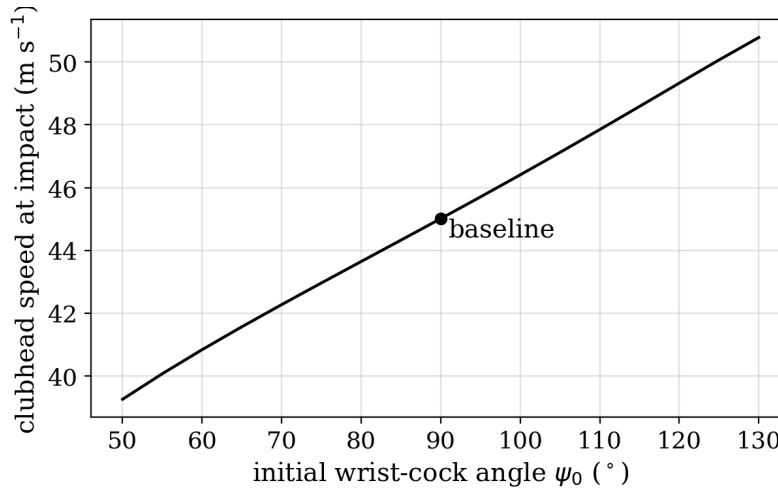


Figure 7: Clubhead speed at impact against the initial wrist-cock angle ψ_0 , using the natural release. The speed increases monotonically from 39.3 m s^{-1} at $\psi_0 = 50^\circ$ to 50.8 m s^{-1} at $\psi_0 = 130^\circ$.

Increasing the wrist-cock from 50° to 130° raises the impact speed from 39.3 to 50.8 m s^{-1} , an average of roughly 0.14 m s^{-1} per degree, and also brings the natural release earlier (from $\delta_n = 75.6^\circ$ down to 37.6°). Both effects follow from the stage 1 equation of motion: a larger ψ_0 reduces I_{rig} , so the same hub torque spins the system up faster, and the club also has a larger angle through which to unwind and gather speed. This is the same efficiency mechanism identified by White, who found that an acutely cocked swing extracts far more speed from the same body effort than a straight-armed one (White, 2006). However, ψ_0 is limited in practice by wrist flexibility, with skilled golfers typically managing 90° – 120° .

Effect of the Hub Torque

Finally, **Table 2** shows the effect of varying the hub torque with all other baseline parameters fixed.

Table 2: Effect of hub torque on the baseline swing (natural release).

τ_0 (N m)	Speed (m s ⁻¹)	Speed (mph)	Downswing time (ms)
80	41.1	91.9	375
90	43.1	96.4	356

τ_0 (N m)	Speed (m s ⁻¹)	Speed (mph)	Downswing time (ms)
100	45.0	100.7	339
110	46.9	104.8	325
120	48.6	108.8	312

The gains are sub-proportional: a 5% increase in torque (from 100 to 105 N m) produces only a 2.07% increase in clubhead speed. This is expected from the work-energy principle, as the kinetic energy at impact is approximately proportional to τ_0 , so the speed scales roughly with $\sqrt{\tau_0}$, which predicts a 2.5% gain; the measured value is slightly lower again because gravity's contribution to the energy is fixed and does not scale with the torque. Swinging "harder" therefore gives diminishing returns compared to the gains available from release timing and wrist-cock, which do not require any increase in the hub torque.

Sensitivity of the Results to the Model Parameters

The conclusions above depend on the baseline parameter values in **Table 1**, several of which (particularly the effective arm mass) are approximate. To test whether the delayed-release gain is a robust feature of the model or just a result of these particular choices, each parameter was varied one at a time by a generous margin and the full release sweep was repeated. For each case, **Table 3** reports the natural release angle, the clubhead speed at the natural release, the best delayed release angle and the resulting maximum speed and gain. The optimum was searched over the delayed region $\delta_r \geq \delta_n$ only, since the early-release branch relies on over-cocking (see the Effect of Release Timing sub-section).

Table 3: Sensitivity of the delayed-release gain to the model parameters, with one parameter changed at a time from the baseline.

Case	Natural release (°)	Speed at natural release (m s ⁻¹)	Best delayed release (°)	Maximum speed (m s ⁻¹)	Gain (%)
Baseline	46.1	45.0	117	47.0	4.4
Arm mass 2.4 kg (-20%)	45.4	46.7	123	49.3	5.6
Arm mass 3.6 kg (+20%)	46.8	43.5	114	45.1	3.6
Hub torque 80 N m (-20%)	47.3	41.1	117	42.8	4.1
Hub torque 120 N m (+20%)	45.3	48.6	120	50.9	4.7
Wrist-cock 70°	56.9	42.3	123	43.7	3.3
Wrist-cock 110°	39.8	47.9	111	49.6	3.7

Case	Natural release (°)	Speed at natural release (m s ⁻¹)	Best delayed release (°)	Maximum speed (m s ⁻¹)	Gain (%)
Backswing angle 150°	43.8	43.7	105	45.0	3.0
Backswing angle 180°	48.1	45.9	132	48.7	6.0

Across all of these variations, the gain from delaying the release stays between 3.0% and 6.0%, and the optimum release angle stays between 105° and 132°. This means that the baseline figure of 4.4% is not a special feature of the chosen parameters: the conclusion that holding the lag well past the natural release produces a speed gain of a few percent holds across the whole realistic parameter range. The size of the gain is most sensitive to the backswing angle θ_0 and the arm mass, and least sensitive to the hub torque and the initial wrist-cock angle.

DISCUSSION AND EVALUATION

Comparison with Published Research

The implementation was first verified numerically using the step-halving and energy checks described in the Numerical Method section. The model itself can then be validated by comparing its predictions against published values, which is done in **Table 4** below.

Table 4: Comparison of the model's predictions with published values. The published speed range runs from the measured male amateur average (93.4 mph) to the professional tour average (≈ 115 mph).

Quantity	This model	Published value	Source
Natural release angle (°)	46.1°	$\approx 47^\circ$	(Jorgensen, 1970; Penner, 2003)
Clubhead speed at impact	45.0 m s ⁻¹	41.8–51.4 m s ⁻¹	(Golf.com, 2023)
Comparable simulated baseline	45.0 m s ⁻¹	44 m s ⁻¹	(Sprigings & Neal, 2000)
Gain from delayed release	+4.4%	$\approx +3\%$	(Pickering & Vickers, 1999)
Speed gain from +5% torque	+2.07%	+1.7%	(Jorgensen, 1999)
Gravity's share of impact energy	12%	13%	(Jorgensen, 1999)
Downswing duration	339 ms	$\approx 200\text{--}300$ ms	(Penner, 2003)

The angles, speeds and energy shares agree with the published values to within a few percent, and the gains from delaying the release and from extra torque have the same size and character as those reported

in the literature. The one systematic discrepancy is the downswing duration, which is somewhat longer than the roughly 0.2 s reported for professional golfers (Penner, 2003). This is most likely because a real golfer's torque is not constant from a standing start: it builds rapidly through the transition, and the hub itself shifts towards the target, both of which shorten the early, slow part of the downswing that dominates the total time. Since these effects act mainly on the timing rather than on the energy transfer, the predicted speeds and trends should be affected far less than the duration.

Physical Interpretation

The results give a clear mechanical explanation for the coaching advice that motivated this investigation. Keeping the wrists cocked holds the club's mass close to the hub, minimising the system's moment of inertia so that the same hub torque produces the largest possible angular acceleration. The energy generated in this compact, fast-rotating state is then transferred to the club during the release. As the club swings outwards, the interaction at the wrists passes momentum and energy down the chain, accelerating the clubhead even as the arms slow down, which is what Miura terms the parametric acceleration of the clubhead (Miura, 2001). "Holding the lag" past the natural release simply extends this storage phase, which is why it produces extra speed and why it requires real physical effort: as **Figure 5** shows, beyond δ_n the wrists must supply a growing torque to resist the club's tendency to swing outwards.

Limitations

The expected effect of each simplification on the results was estimated in the Assumptions and Justification sub-section. The most important limitations, and what a more complete model would add, are:

- **Fixed hub and constant torque:** adding the lateral hub shift and a realistic torque profile would raise the absolute speeds by several percent and shorten the downswing, bringing the predicted duration closer to measured values.
- **Passive release:** allowing an actively driven wrist torque after the release, within biomechanical limits, would add up to 9% more clubhead speed (Sprigings & Neal, 2000), so the speeds reported here are conservative.
- **Rigid shaft:** modelling the deflection of the shaft would slightly change the delivered speed and dynamic loft (Milne & Davis, 1992).
- **Effective arm parameters:** the arm mass and radius are fitted values rather than measured biomechanical quantities. The sensitivity analysis shows that the trends survive a $\pm 20\%$ change in the arm mass, but the absolute speeds should be read with this uncertainty in mind.
- **Speed is not distance:** the model maximises clubhead speed, but an extremely delayed release degrades the impact geometry (see the Effect of Release Timing sub-section), which in reality

would cost accuracy, launch angle and therefore distance. The optimal release for actual driving distance is consequently somewhat earlier than the optimum in **Figure 6**.

CONCLUSION

In conclusion, the double pendulum model shows that the clubhead speed of a driver shot increases as the wrist release is delayed, rising from 45.0 m s^{-1} at the natural release ($\delta_n = 46.1^\circ$) to a maximum of 47.0 m s^{-1} when the release is held until the arm has swung through 118° , a gain of 4.4%. Releasing earlier than the natural point costs almost nothing relative to the natural release, but allowing the wrists to go fully passive from the top is much worse (37.2 m s^{-1}) and physically impossible for human wrists. Larger initial wrist-cock angles and larger hub torques both raise the speed further, although increasing the torque gives diminishing square-root returns. All of these predictions agree well with the published double pendulum literature, from the natural release angle (46.1° vs $\approx 47^\circ$) to the size of the late-hit gain. The sensitivity analysis shows that this conclusion is robust, with the delayed-release gain staying between 3.0% and 6.0% when each model parameter is varied by a generous margin.

The practical answer to my original question is that “holding the lag” is not just a feeling-based instruction but a direct consequence of rotational mechanics. The cocked wrists keep the system’s moment of inertia small while energy is being generated, and the delayed release then transfers this stored energy into the club at the last moment. This means that in my upcoming range sessions, rather than simply trying to swing harder, I should focus on maintaining my wrist-cock deeper into the downswing, since the model suggests this is worth around 4–5 mph of clubhead speed, or up to 12–15 yards of distance (Trackman, n.d.), without needing any increase in hub torque, provided the club face delivery is not compromised in the process.

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APPENDIX: CORE SIMULATION CODE

The core of the Python program used to generate the results in this paper is listed below. Running it reproduces the baseline and delayed-release clubhead speeds from the Results section. The full version, which also produces the figures and the parameter sweeps, is available from the author on request.

```
import numpy as np

# parameters (Table 1)
m1, L1 = 3.0, 0.67
m2, L2, r2, I2 = 0.382, 1.092, 0.661, 0.071
g, tau0 = 9.81, 100.0
A = (m1 + m2) * L1**2
B = m2 * L1 * r2
C = I2 + m2 * r2**2

def rigid_acc(th, psi0):
    # stage 1 rigid-body equation
    Irig = A + C + 2 * B * np.cos(psi0)
    return (-tau0 - (m1 + m2) * g * L1 * np.sin(th)
            - m2 * g * r2 * np.sin(th + psi0)) / Irig

def hold_torque(th, thd, thdd, psi0):
    # wrist torque needed to hold the cock
    return ((C + B * np.cos(psi0)) * thdd
            + B * thd**2 * np.sin(psi0)
            + m2 * g * r2 * np.sin(th + psi0))

def deriv(y):
    # stage 2 coupled equations of motion
    th, thd, ph, phd = y
    cD, sD = np.cos(ph - th), np.sin(ph - th)
    M = np.array([[A, B * cD], [B * cD, C]])
    R = np.array([-tau0 + B * sD * phd**2
                  - (m1 + m2) * g * L1 * np.sin(th),
                  -B * sD * thd**2 - m2 * g * r2 * np.sin(ph)])
    thdd, phdd = np.linalg.solve(M, R)
    return np.array([thd, thdd, phd, phdd])

def rk4(f, y, h):
    # fourth-order Runge-Kutta step
```

```

k1 = f(y); k2 = f(y + h / 2 * k1)
k3 = f(y + h / 2 * k2); k4 = f(y + h * k3)
return y + h / 6 * (k1 + 2 * k2 + 2 * k3 + k4)

def swing(psi0_deg=90.0, release_deg=None, h=1e-5):
    # returns clubhead speed at impact; release_deg=None
    # uses the natural release condition
    psi0 = np.deg2rad(psi0_deg)
    th0 = np.deg2rad(166.0)
    th, thd, t = th0, 0.0, 0.0
    f1 = lambda y: np.array([y[1], rigid_acc(y[0], psi0)])
    prev_tw = None
    while True:
        # stage 1: cock held
        thdd = rigid_acc(th, psi0)
        tw = hold_torque(th, thd, thdd, psi0)
        if release_deg is None:
            if prev_tw is not None and prev_tw < 0 <= tw:
                break
            # natural release
        elif th0 - th >= np.deg2rad(release_deg):
            break
            # prescribed release
        prev_tw = tw
        th, thd = rk4(f1, np.array([th, thd]), h)
        t += h
    y = np.array([th, thd, th + psi0, thd])
    while True:
        # stage 2: free hinge
        yp = y.copy()
        y = rk4(deriv, y, h)
        t += h
        if yp[2] > 0.0 >= y[2]:
            # club vertical: impact
            frac = yp[2] / (yp[2] - y[2])
            yi = yp + frac * (y - yp)
            vx = (L1 * yi[1] * np.cos(yi[0])
                  + L2 * yi[3] * np.cos(yi[2]))
            vy = (L1 * yi[1] * np.sin(yi[0])
                  + L2 * yi[3] * np.sin(yi[2]))
            return np.hypot(vx, vy)

print(round(swing(), 2))          # baseline: 45.01 m/s
print(round(swing(release_deg=118.0), 2))  # late hit: 47.01

```