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Detecting Market Crashes and Structural Changes in Small-Scale Baltic Financial Markets using Topological Data Analysis

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ABSTRACT

Stock market crashes are large declines in stock prices across major sectors, leading to a large decrease in asset values, household wealth and consumer confidence. During market crashes, the structure of the market undergoes significant changes, which can be difficult to detect using traditional financial indicators.

Topological Data Analysis (TDA) is a mathematical approach that examines the structure of datasets by representing data as geometric shapes and analysing how these shapes change. Topological Data Analysis (TDA) has recently become a popular tool for detecting changes in the underlying structure of the market and therefore serving as an indicator of financial crises. However, no research has been conducted on the application of this approach to small-scale, low-liquidity markets, characterized by a limited number of active buyers and sellers, lower trading volumes and sharper price movements.

Therefore, this paper examines the applicability of the TDA framework to small-scale financial markets, using the Baltic market as a case study and using 5 major Baltic indices.

The results showed that the proposed TDA methodology detected three distinct periods of market crises in the Baltic market. The large declines in index values coincided with topological metric signals, also enabling to assess the duration of each market crash's effect on market structure. These findings support the use of TDA in low-liquidity markets and show its potential as an additional tool for monitoring and detecting market instability.

INTRODUCTION

Market crises are generally unexpected and often have a significant impact on the economy. Researchers worldwide have long sought to predict when and how market crashes will occur. Traditional methods of predicting these price shifts, such as volatility-based models, often have several limitations: they typically react to market changes only after a crisis has occurred, they are highly sensitive to noise and fail to detect non-linear complex interactions and hidden structural changes. In recent years, with the advanced

development of machine learning, new ways of shape recognition have emerged, and one of them is Topological Data Analysis.

Existing studies have primarily focused on large financial markets, such as the United States, China, Singapore and Taiwan (Rai et al., 2024; Yen & Cheong, 2021), and, based on a review of the existing literature, no research has investigated the application of TDA to small-scale financial markets. In this context, a small-scale market refers to a localized economic and trading environment characterized by relatively low production volumes, limited capital investment and a small number of local buyers and sellers. Our initial thought was that this might be due to the structural nature of such markets, which often lack volume and exhibit high levels of noise, potentially making it more challenging to identify topological structures using TDA.

Therefore, this study addresses the research problem of evaluating the applicability of TDA in small-scale financial markets for detecting market crashes and structural changes, using the Baltic financial market as a case study.
Our hypothesis is that TDA can still identify different market states in small-scale markets and may serve as an indicator of structural changes and market crashes.

RELATED WORK

A systematic review of applications of TDA in financial crisis detection revealed that no prior research has investigated the applicability of topological data analysis to small-scale financial markets, highlighting a gap in the literature. Moreover, a study examining whether the Baltic market is efficient showed that widely applied market-analysis frameworks do not always produce the same results in the Baltic market, therefore highlighting the need to investigate the applicability of different frameworks to small-scale markets.

1. Topological Machine Learning for Financial Crisis Detection: Early Warning Signals from Persistent Homology

In a study by Guritanu et al. (2025), the authors proposed a strictly causal early-warning framework for financial crises based on Topological Data Analysis (TDA). The framework consisted of several consecutive steps: sliding windows of daily log-returns were embedded as point clouds, Vietoris–Rips persistent homology was computed, and the resulting persistence diagrams were summarised using persistence landscapes, whose norm was then processed through a detection rule based on sudden increases, short-term deviations, and sustained elevation. This study focused on four major U.S. indices, specifically S&P 500, NASDAQ, DJIA, Russell 2000. The framework identified two of the four canonical crises and produced a signal around the 2008 financial crisis, achieving a mean lead time of approximately 34 days. Although their study focused on early-warning detection, whereas our work investigates the applicability of TDA for detecting structural changes in financial markets, it nevertheless

demonstrates the potential of TDA to detect major changes in financial market structure and provides support for our study.

2. Using Topological Data Analysis (TDA) and Persistent Homology to Analyze the Stock Markets in Singapore and Taiwan

Yen et al. (2021) used TDA to investigate topological changes associated with crashes in the Singaporean and Taiwanese stock markets and supported their hypothesis that TDA can effectively capture these changes. They used several TDA frameworks, including Betti numbers, Euler characteristics, barcodes, persistence diagrams and persistence landscapes, showing that these frameworks can detect changes in the underlying structure of financial markets concurrent with the emergence of market crashes.

They also used toy models, which illustrate three different sequences of topological changes, to demonstrate that, when a market crash emerges, the cross-correlations between stocks remain concentrated in a single large cluster, and only over time, this cluster begins to break apart as handles disappear, forming several distinct smaller clusters. Most importantly, they showed that during financial crises, the persistence diagrams of different dimensions change, with points becoming closer to the diagonal, which corresponds to a shorter life expectancy for the homology groups. However, since persistence diagrams act only as an indicator of topological changes in the structure of the data, it should be noted that those changes are not unified and homology groups can also become more persistent during market crises. Moreover, they explained that TDA is robust in deciding which topological scale to use for data analysis. For that reason, to determine which range allows the topological structure to be the most persistent, they scanned through a range of correlation threshold values. They stated that TDA homologies are robust to random noise, reducing the accumulation of noise in the data and the risk of overfitting**.**

3. Identifying Extreme Events in the Stock Market: A Topological Data Analysis

Among the prior literature, the framework proposed by Rai et al. (2024) serves as the primary theoretical foundation for our work. While our study adapts their methodology to small-scale markets, their paper employs TDA to detect extreme events in the stock market at a continental level. They analyzed index crashes during the 2008 financial crisis and the COVID-19 pandemic. In their study, they proposed two different models for identifying extreme events in the market: Wasserstein distance and L^1 and L^2 norms. Most importantly, they showed that during market crashes, the Wasserstein distance of the world's leading indices rises abruptly, surpassing a threshold of $\mu_W + 4\sigma$. It is important to note that this threshold is not unified, and for each market, several thresholds should be examined to understand which one should be applied. For instance, after the occurrence of EEs, they observed strong crashes surpassing $\mu_W + 2\sigma$ for an extended period in the banking sector. Another important point is that they showed TDA to be successful in identifying the duration of shocks after the occurrence of EEs.

4. Market efficiency of Baltic stock markets: A fractional integration approach

In their study, Gil-Alana et al. (2018) investigated whether the Baltic stock market is efficient by analysing data from the Baltic Benchmark Gross (OMXBBGI), OMX Tallinn (OMXT), OMX Riga (OMXR), and OMX Vilnius (OMXV) indices. Their findings showed that the Baltic stock market does not follow the random walk framework and exhibits long-memory behaviour. They noted that the Baltic market is relatively small-scale and new, with low market capitalisation and trading volumes compared to the larger markets that are typically analysed within this framework. Although the aim and methodology of their study differ from those of our research - they used fractional integration methods to examine market efficiency - their findings are relevant to our study because they demonstrate that the statistical properties of small-scale and low-liquidity financial markets may differ from those observed in larger markets. Therefore, methods and hypotheses that are effective for large financial markets should not necessarily be assumed to produce the same results when applied to smaller markets, which provides further justification for investigating the applicability of TDA and other market-analysis frameworks to small-scale financial markets.

METHODOLOGY

Data collection and cleaning

The historical price data for the Baltic stock market indices were collected from the Nasdaq Baltic website (Nasdaq, n.d.). The selected indices were the OMX Baltic Benchmark GI, OMX Baltic 10, OMX Riga GI, OMX Tallinn GI and OMX Vilnius GI. These indices were chosen because they are the primary Baltic indices and represent the Baltic stock market as well as the individual stock exchanges of Latvia, Estonia, and Lithuania. The collected data were then cleaned and standardized, as national holidays and non-trading days vary across the Baltic markets. Specifically, the dates for all five indices were aligned, and dates on which data were not available for all indices were removed from the dataset. This study utilizes the Python libraries ripser, NumPy and persim for the implementation of the TDA.

Logarithmic Returns and Vector Representation

Before applying TDA, the index price data was transformed into logarithmic returns in Python using the NumPy library. Logarithmic returns were used instead of simple returns because they add up over time, are less sensitive to large fluctuations and provide a more realistic measure of investment performance (Matabaro, 2025).

For each index $i \in \{1, 2, 3, 4, 5\}$ on trading day t, the logarithmic return $r_{i,t}$ was calculated as:

$$r_{i,t} = ln(\frac{P_{i,t}}{P_{i,t-1}})$$

Where $P_{i,t}$ represents the index price at time t and $P_{i,t-1}$ represents the index price from the previous trading day.

To capture the topological state of the Baltic market, the logarithmic returns of all five indices for a given day t were combined into a single five-dimensional vector:

$$r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \\ r_{3,t} \\ r_{4,t} \\ r_{5,t} \end{bmatrix}$$

where each coordinate of r_t corresponds to the logarithmic return of one of the five market indices (OMX Baltic Benchmark GI, OMX Baltic 10, OMX Tallinn GI, OMX Riga GI, and OMX Vilnius GI).

Rolling window approach

Point clouds were constructed using sliding window approach. For a given starting trading day t, two consecutive non-overlapping windows of size w are defined as:

Window 1 ($X_{1,t}$) :

$$X_{1,t} = \{r_t, r_{t+1}, \dots\dots, r_{t+(w-1)}\}$$

containing w consecutive daily return vectors.

Window 1 ($X_{2,t}$) :

$$X_{2,t} = \{r_{t+w}, r_{t+w+1}, \dots\dots, r_{t+(2w-1)}\}$$

containing the following w daily return vectors.

Thus, Window 2 begins immediately after the last observation of Window 1. The windows then slide forward by one trading day, so the next pair of windows becomes:

$$X_{1,t+1} = \{r_{t+1}, r_{t+2}, \dots\dots, r_{t+w}\}$$

and

$$X_{2,t+1} = \{r_{t+1+w}, r_{t+2+w}, \dots\dots, r_{t+2w}\}$$

This process continues throughout the dataset.

Vietoris–Rips complex

From each point cloud X_t a Vietoris–Rips complex was constructed. The Vietoris–Rips complex is a sequence of simplicial complexes that depends on a scale parameter (or filtration parameter) $\epsilon \geq 0$. A simplicial complex is a generalization of a graph that includes higher-dimensional simplices: triangles (2-simplices), tetrahedra (3-simplices), etc.

Definition. Let S be a finite set of points in some Euclidean space and $r \geq 0$. The Vietoris-Rips complex of S at scale r is:

$$Vietoris - Rips(r) = \{\sigma \subseteq S \mid diam\, \sigma \leq 2r\}.$$

Where $diam(\sigma)$ denotes the diameter of the simplex σ. Equivalently, the Vietoris–Rips complex consists of all subsets of S whose diameter is at most $2r$ (Edelsbrunner & Harer, 2010).

As r increases, simplices are added to the complex, changing its topology (new connected components, cycles, voids appear).
In each 50-day window, after the Vietoris–Rips complex is formed, it is used to compute persistent homology for both dimension H_0, representing the clustering and connectivity of the market indices, and dimension H_1, capturing the multi-dimensional loops and structural voids. The window then slides forward by one trading day, and the entire process is repeated to monitor how both H_0 and H_1 features evolve.

For this study, only H_0 and H_1 homologies were considered, as these homology dimensions have been used in previous studies applying persistent homology to financial crisis detection (Guritanu et al., 2025). Furthermore, when H_2 was examined, its persistence points were concentrated along the diagonal during both crisis and non-crisis periods (see Appendix C). This means that H_2 features had low persistence and provided limited information needed for distinguishing between crisis and non-crisis periods. For that reason, H_2 was excluded from the study.

Persistence Diagrams

Persistent homology is a method within TDA used to study topological features of the data, such as shape and structural properties, across multiple scales.

Mathematically, persistent homology can be captured by a diagram called persistence diagram. A persistence diagram is a 2-dimensional plot where each point represents a specific topological feature. Each point is defined by birth (b) and death (d) coordinates, where birth is the scale of filtration value at

which a topological feature first appears and death is the scale at which the feature either disappears or merges with another (Oseko et al., 2024).

This study focused on two types of topological features:

H_0 (Dimension 0) - measures the connectivity of the data. Each point in H_0 represents separate points or clusters in the data.

H_1 (dimension 1) - measures 1-dimensional holes, representing gaps where data points are missing in the center of a structure.

The distance between the birth and death points shows for how long a feature stayed as the scale changed and is calculated as d - b.

When analysing persistence diagrams, we need to look at the position of the points relative to the diagonal line y = x. Points located far from the diagonal line represent features that persist over many scales (there is a large distance between when the feature was born and when it died) and is very likely a real part of the data's shape. Conversely, points located near the diagonal line represent features that die almost immediately and are considered as noise.

Periods of relative market stability generally exhibit fewer persistent features (there are more points located near the diagonal line). In contrast, during periods of instability, the market structure changes significantly, producing more long-lived topological structures (there are more points located further from the diagonal line).

In this research, persistence diagrams were generated from consecutive 50-day sliding windows of logarithmic return data.

Two time periods were used:

June 2019 to September 2019 - this period is an example of a stable market, since it covers the period after the global market corrections of 2018, when markets stabilized after the US-China trade tensions, and it also precedes the COVID-19 pandemic.

February 2022 to June 2022 - this period is an example of an unstable market, as it coincides with the Russian invasion of Ukraine and the subsequent economic uncertainty in the Baltics region.

Wasserstein Distance

The Wasserstein metric represents the minimal cost required to transform one probability distribution into another (Leo et al., 2023). In this study, the Wasserstein distance is used to measure the difference between two persistence diagrams, indicating how much one diagram must be transformed to match the other.
Wasserstein distance is defined as:

$$W_p(X,Y) = \inf_{\gamma \in \Gamma(X,Y)} \left(\sum_{x \in X} \|x - \gamma(x)\|^p \right)^{1/p}$$

Where X and Y are persistence diagrams, $\Gamma(X,Y)$ represents all possible matchings between their points, and $\gamma(x)$ denotes the matched feature corresponding to point x. Larger Wasserstein distances indicate stronger topological changes between neighboring time periods and correspond to periods of increased market instability.

To calculate the Wasserstein Distance in the Baltic market, the persim.wasserstein() function from the persim Python library was used. The date assigned to the resulting Wasserstein distance value corresponds to the end date of the second window of log returns being compared. Thus, the assigned date corresponds to the endpoint of the data used to construct the second persistence diagram. The Wasserstein distance was computed for the H_0 dimension to understand whether the market structure changed and became fragmented during periods of market crisis.

The data collected covers the period from April 1, 2016, to May 26, 2026. The reason for choosing this timeframe was to include both relatively stable market periods and crisis periods, such as the COVID-19 market crash and the Russian invasion of Ukraine in 2022. To identify periods of unusual topological changes in the market, a threshold value was introduced. After testing several threshold values, the threshold was defined as $\mu_W + 1\sigma$, where σ is the standard deviation and μ_W is the average value of the Wasserstein Distance. This threshold value was chosen because it identifies unusually high Wasserstein distance values and is robust to random noise.

Persistence Landscapes and Their L^1 and L^2 Norms

Another way of analysing persistence diagrams is through persistence landscapes and their norms. To construct a persistence landscape from a persistence diagram P_k, each birth-death point $(b_a, d_a) \in P_k$ is first transformed into a piecewise-linear function $f_{(b_a, d_a)}(x)$:

$$f_{b_\alpha,d_\alpha}(x) = \begin{cases} x - b_\alpha, & \text{if } x \in \left(b_\alpha, \frac{b_\alpha+d_\alpha}{2}\right]; \\ -x + d_\alpha, & \text{if } x \in \left(\frac{b_\alpha+d_\alpha}{2}, d_\alpha\right); \\ 0, & \text{if } x \notin (b_\alpha, d_\alpha). \end{cases}$$

This function forms a triangular shape whose base is determined by the birth and death values, b_a and d_a. Its maximum occurs at the midpoint $(b_a + d_a)/2$. The persistence landscape is then defined as a sequence of functions $\lambda = (\lambda_k)_{k\geq 1}$, where

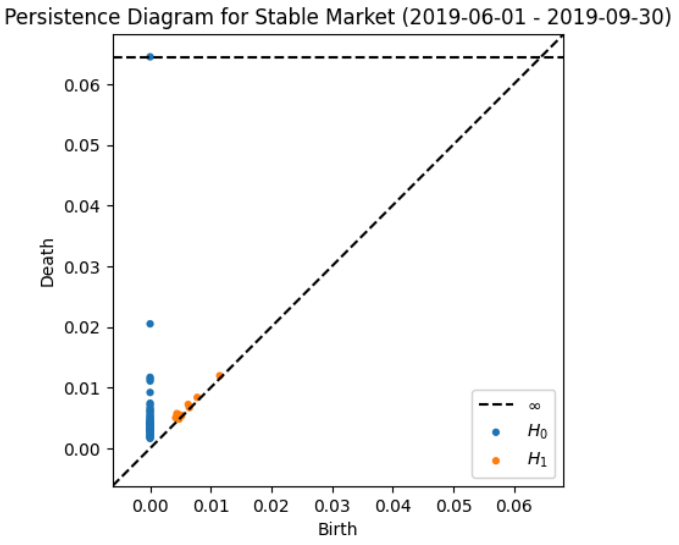
$$\lambda_k(x) = k - \max\{f(b_a d_a)(x) \mid (b_a d_a) \in P_k\}$$

Here, the k-max denotes the k-th largest value among the functions at a given x. Thus, $\lambda_1(x)$ represents the largest value, $\lambda_2(x)$ the second largest, and so on. In this study, the $L^1$and L^2 norms were used. For a persistence landscape λ, the general L^p-norm is defined as

$$\|\lambda\|_p = \left(\sum_{k=1}^{\infty} \int_{\mathbb{R}} |\lambda_k(x)|^p \, dx\right)^{\frac{1}{p}}.$$

RESULTS

Persistence Diagrams of Baltic Market

<table>
<tr>
<td>
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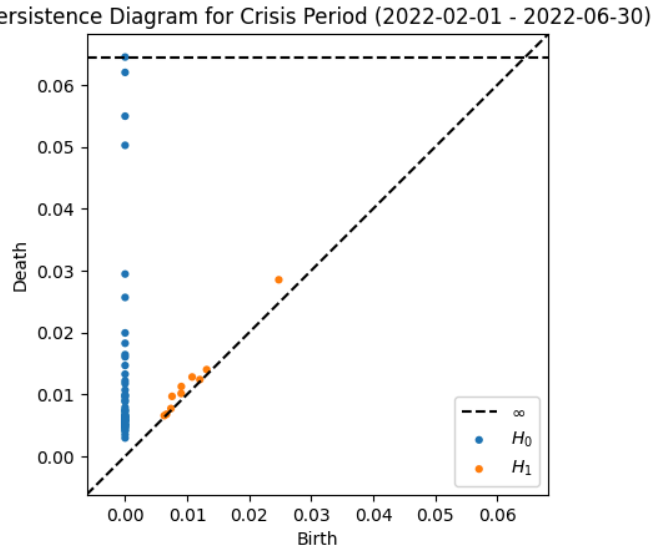
<td>
</td>
</tr>
<tr>
<td>Stable Market (2019.06.01 to 2019.09.30)

Number of points in dimension 0 (H_0): 50

Number of points in dimension 1(H_1): 13</td>
<td>Crisis Period (2022.02.01 to 2022.06.30)

Number of points in dimension 0 (H_0): 50

Number of points in dimension 1(H_1): 10</td>
</tr>
</table>

Figure 1: Comparison of Persistence Diagrams for the Baltic Market during Stable (2019) and Crisis (2022) periods.

In Figure 1, the blue points represent 0-dimensional features H_0, while the orange points represent 1-dimensional features H_1. The dashed line corresponds to the diagonal line of the persistence diagram. During the stable market period, points are generally located closer to the diagonal line, indicating low-persistence structures. In contrast, during the crisis period, more points are located farther from the diagonal, particularly H_0 features. This suggests the emergence of more persistent topological structures.

Wasserstein Distance of the Baltic Market

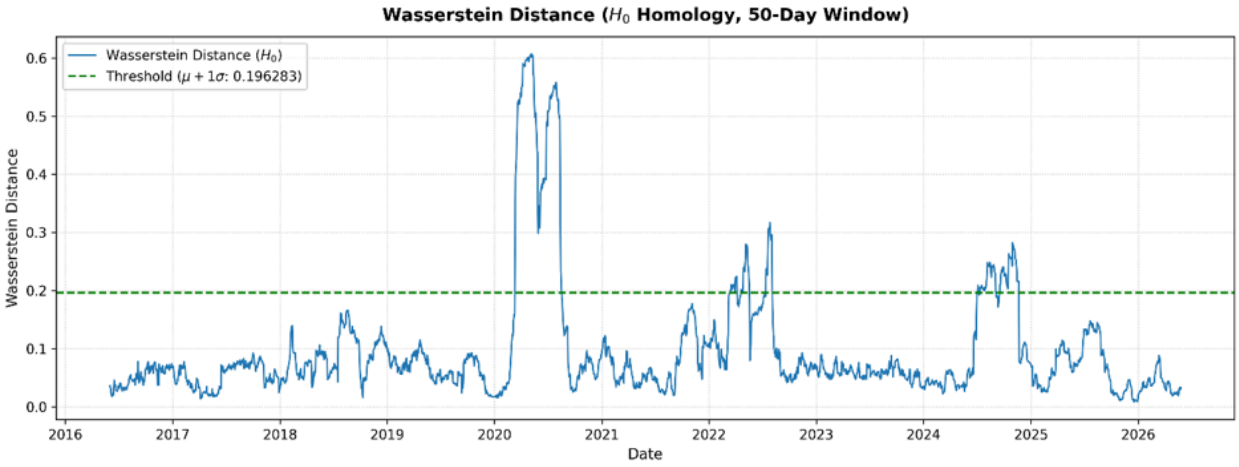


Figure 2: Dynamics of the Wasserstein Distance for Baltic Market Indices (H_0 Persistence Diagrams, 50-Day Windows, 04.01.2016 – 26.05.2026)

Start date	End date	Calendar days	Trading days	Peak wasserstein	Start date
12.03.2020	09.04.2020	29	21	0.590400	12.03.2020
14.04.2020	30.04.2020	17	13	0.601734	14.04.2020
05.05.2020	19.06.2020	46	33	0.606679	05.05.2020
25.06.2020	14.08.2020	51	36	0.557922	25.06.2020
14.03.2022	06.04.2022	24	18	0.224312	14.03.2022
20.04.2022	18.05.2022	29	20	0.279337	20.04.2022

13.07.2022	04.08.2022	23	17	0.316797	13.07.2022
04.07.2024	05.07.2024	2	2	0.208988	04.07.2024
11.07.2024	03.09.2024	55	37	0.248067	11.07.2024
17.09.2024	21.11.2024	66	46	0.281791	17.09.2024

Table 1: Threshold Exceedance Intervals for Wasserstein Distance

Figure 2 illustrates the dynamics of the Wasserstein Distance over the period from January 4, 2016, to May 26, 2026 (H_0 persistence diagrams, 50-day sliding window). The blue line represents the Wasserstein Distance values, while green dashed horizontal line represents threshold value defined at $\mu_W + 1\sigma$. The periods with the highest values in the Wasserstein distance occur in 2020, 2022 and 2024. Table 1, accordingly, provides time periods when the Wasserstein distance surpassed the threshold and the corresponding highest values of the distance for each period.

Persistence Landscapes of Baltic Market

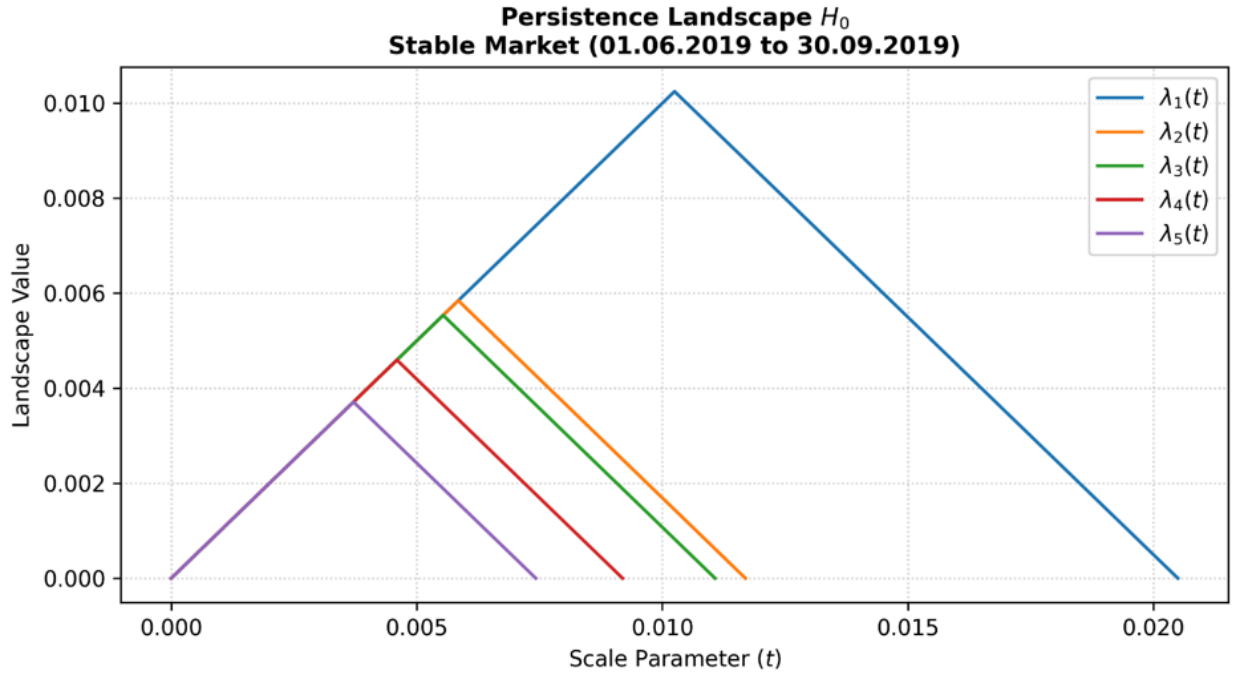


Figure 3: Persistence Landscape for the Baltic Market during Stable period (01.06.2019 to 30.09.2019)

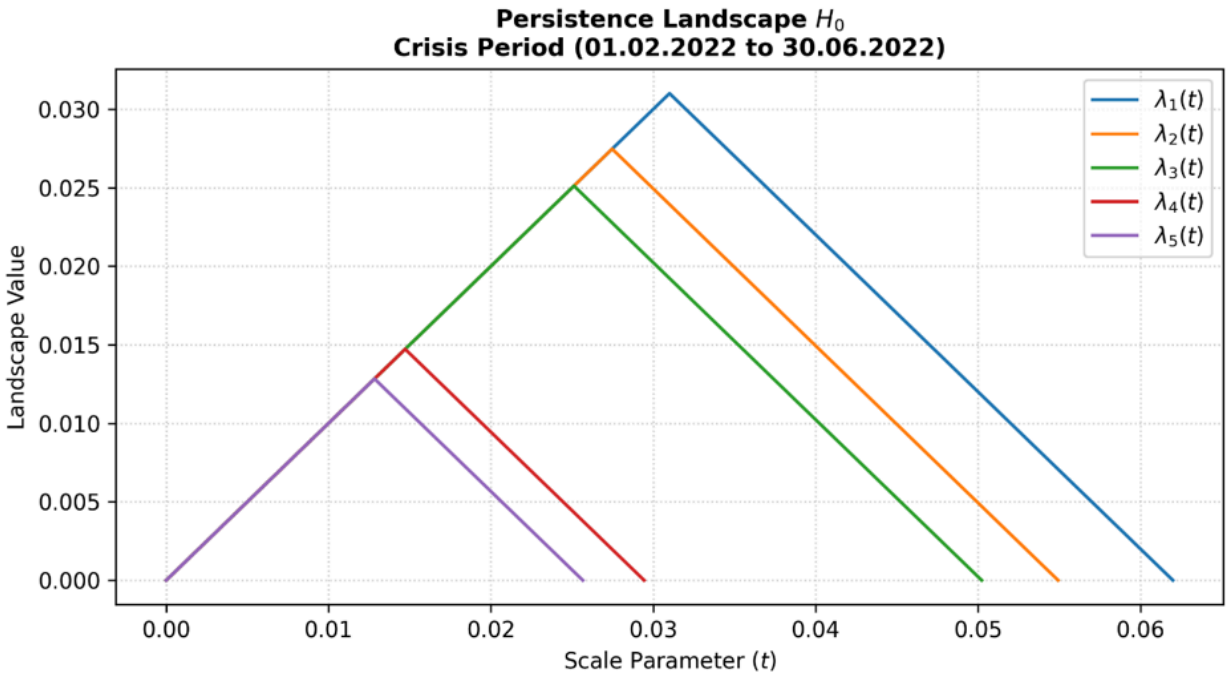


Figure 4: Persistence Landscape for the Baltic Market during Crisis Period (01.02.2022 to 30.06.2022)

Figure 3 and Figure 4 illustrate the persistence landscapes for the Baltic market during stable and crisis periods, respectively. The blue line represents the first layer of the persistence landscape, the yellow line the second, the green line the third, the red line the fourth and the violet line the fifth. The maximum values of each persistence-landscape layer during the crisis periods exceed the corresponding maximum values during the non-crisis periods. This indicates that, during market crises, H_0 topological features become more persistent and remain disconnected over a longer range of the Vietoris-Rips filtration parameter.

Persistence Landscapes' Norms of Baltic Market

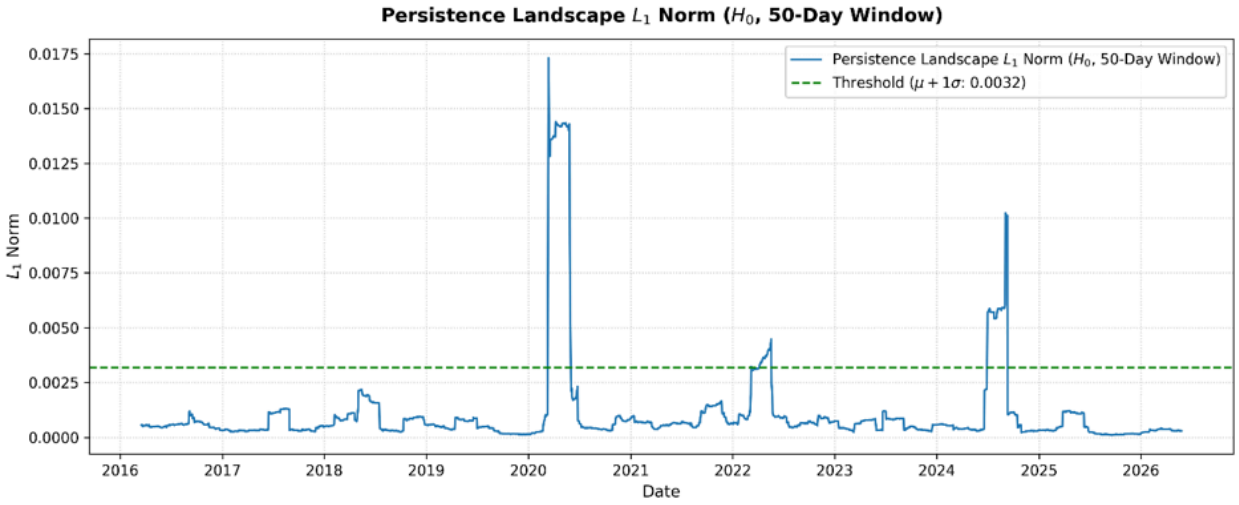


Figure 5: Dynamics of the L^1 Norm of Persistence Landscapes for Baltic Market Indices (04.01.2016 - 26.05.2026)

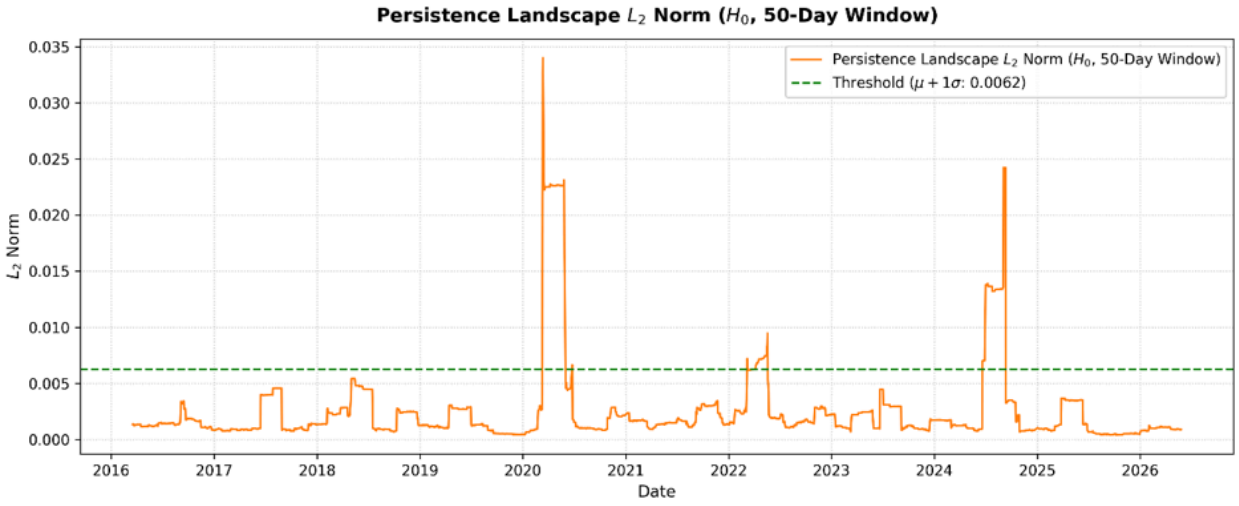


Figure 6: Dynamics of the L^2 Norm of Persistence Landscapes for Baltic Market Indices (04.01.2016 - 26.05.2026)

Norm	Start Date	End Date	Duration (Days)	Peak Value
L^1 Norm	12.03.2020	29.05.2020	52	0.01730
	08.03.2022	08.03.2022	1	0.00323
	06.04.2022	19.05.2022	29	0.00448
	01.07.2024	10.09.2024	50	0.01024
L^2 Norm	12.03.2020	29.05.2020	52	0.03399
	25.06.2020	25.06.2020	1	0.00664
	08.03.2022	08.03.2022	1	0.00720
	24.03.2022	28.03.2022	3	0.00625
	01.04.2022	19.05.2022	32	0.00946
	20.06.2024	10.09.2024	56	0.02424

Table 2: Threshold Exceedance Intervals for L^1 and L^2 Norms

Figure 5 and Figure 6 illustrate the dynamics of the L^1 and L^2 norms of the persistence landscapes, respectively. The blue line represents the L^1 norm, and the orange line represents the L^2 norm. The green dashed horizontal line indicates the $\mu_W + 1\sigma$ threshold. Both norms exhibit threshold exceedances during 2020, 2022 and 2024, successfully capturing major and local market crises. Additionally, Table 1 illustrates time intervals when the L^1 and L^2 norms exceeded this threshold and peak value recorded for each period.

DISCUSSION

As shown in Figure 1, the crisis period (01.02.2022 to 30.06.2022) exhibits H_0 features with significantly higher persistence values (located further from the diagonal line) compared to the stable market period (01.06.2019 to 30.09.2019). In persistent homology, points that persist longer (i.e., have a larger distance between birth and death values) represent topological features that remain stable across a wider range of filtration values. Previous studies have also reported changes in H_0 persistence diagrams during periods of financial crisis and interpreted these changes as evidence of a reorganization of the market's topological structure. For instance, Guritanu et al. (2025) also observed more persistent H_0 features during turbulent market periods. At the same time, the number of H_1 features decrease from 13 during the stable period to 10 during the crisis period. However, the crisis period also contains one relatively persistent H_1 feature. Together, these observations suggest that the topological structure of the Baltic financial market changes during periods of financial stress and that persistence diagrams are effective in capturing these structural changes.

From Figure 2, the three periods with the highest peaks in the Wasserstein distance are 2020, 2022, and 2024. During all of these periods, significant market crises or episodes of heightened market instability occurred.
In 2020, financial markets in the Baltic region were significantly affected by the COVID-19. According to Latvijas Banka (2021), the COVID-19 economic shock negatively affected the Latvian banking sector and broader financial markets, increasing instability in financial asset prices during 2020. Moreover, restrictions introduced during the pandemic disrupted trade, business activity and consumer demand across the Baltic states, which contributed to heightened uncertainty in financial markets.

Several spikes also occurred during the period from March to July 2022, which coincided with the beginning of the war in Ukraine. Prior to the invasion of Ukraine, the Wasserstein Distance of the Baltic market remained low, ranging between 0.05 and 0.10. On 23 February 2022, the metric was 0,063, after which it surged to 0,325 by 12 March 2022, surpassing the established threshold. Over these 17 days, the Wasserstein Distance increased by approximately 516%. The Wasserstein Distance remained above the threshold until 4 August 2022. Accordingly, major structural changes in the Baltic market persisted for approximately five months following the beginning of the war in Ukraine. According to the International Monetary Fund (2022), prior to the war, Russia was the primary supplier of natural gas to the Baltic region. The spillover effects from the war manifested in spikes in energy and food prices, supply disruptions, refugee inflows, and increased budgetary pressures. In May, inflation accelerated to 16.8%, driven largely by surging energy and food prices. These created economic uncertainty and market instability in the Baltic region, which evolved into a broader economic crisis across the region and, as shown in Figure 2, changed the structure of the Baltic market.

Even though there were no major market crashes in the Baltic market during 2024, the peak in the Wasserstein distance is attributed to Nasdaq Riga's decision to approve JSC "Latvijas Gāze"'s application to delist its bearer shares from the Baltic Secondary List. The decision was adopted on 19 June 2024, while 23 August 2024 was designated as the last trading day for the company's shares (Latvijas Gāze, n.d.).

By comparing the Wasserstein Distance with the timeline of these events, we concluded that the delisting was the most likely cause of the structural changes observed in the market. On 19 June 2024, the day Nasdaq Riga approved JSC "Latvijas Gāze"'s delisting application, the Wasserstein Distance was 0.056, but it increased to 0.113 on the following day. The metric continued to rise, and on 4 July 2024, it exceeded the threshold. The Wasserstein Distance reached its highest value of 0.248 on 14 August 2024, nine days before the last trading day of the company's shares. Afterward, the Wasserstein Distance remained above the threshold until 21 November 2024, indicating that the structural changes in the market persisted for approximately six months.

It is important to understand what happened in the market during that time. The approved delisting of JSC "Latvijas Gāze" was followed by a decline in the OMX Riga GI index, from 1,251.3 closing value on 19 June 2024 (the day Nasdaq Riga approved JSC "Latvijas Gāze"'s delisting application) to 843.08 on 6 August 2024, which is a 32.63%. decline. Since then, the index has remained within the range of approximately 850–900. Concurrently, other indices maintained their usual price ranges and did not exhibit unusual price fluctuations. This difference in price movements may have contributed to the observed changes in market structure and produced a crisis signal.

The JSC "Latvijas Gāze" is one of the leader companies of the Baltic energy sector and has been listed on the NASDAQ Riga stock exchange since February 15, 1999. It provides wholesale and retail natural gas services to business customers in Latvia, Estonia, Lithuania and Finland, while in the household segment, JSC "Latvijas Gāze" is the largest and most prominent natural gas trader in Latvia (Joksta, 2024). In connection with the planned delisting, the Supervisory Committee approved a mandatory share buyback at €3.35 per share (Latvijas Gāze, 2024). Given the company’s total share capital of 39.9 million shares (Latvijas Gāze, 2025), this valued the company at approximately €133.7 million at the time of delisting. Following the buyback, only 5,553 treasury shares remained (Joksta, 2024).

Particularly interesting was the finding that such substantial changes in the market structure were caused by the delisting of the shares of a single company, which would be unusual in larger markets. This suggests that small markets are more interconnected and that changes affecting a single listed company can have a significant impact on the overall market structure. Hence, we concluded that the Wasserstein distance can serve as a tool not only for detecting global crises but also for detecting local events that affect the market structure.

The analysis of Persistence Landscapes during stable and crisis market periods (Figures 3 and 4) was consistent with the results of the persistence diagrams, illustrating that each layer became more persistent during a market crisis. Specifically, both the range of the scale parameter t and the maximum landscape value increased across all layers.

The L^1 norm exhibited results preceding the Wasserstein distance. For instance, it detected the war-related 2022 crisis on March 8, 2022, while the Wasserstein distance detected it 4 trading days later, on March 14, 2022. Similarly, it detected the 2024 Riga market crisis on June 1, 2024, 12 days before the Wasserstein distance. However, the threshold exceedance occurred concurrently with the price fall, so it did not provide a predictive signal. Nonetheless, the Wasserstein distance values remained above the threshold for longer periods. For instance, the L^1 norm indicated the end of the 2022 crisis on May 19, 2022, whereas the Wasserstein distance remained above the threshold until August 4, 2022. Moreover, the 2024 crisis faded faster under the L^1 norm: in terms of the Wasserstein distance, the crisis lasted until November 21, 2024, compared to September 10, 2024, for the L^1 norm.

L^1 and L^2 norms showed concurrent results for the end of the crises. However, in the case of the 2024 crisis, the L^2 norm detected the event much faster, on June 20, 2024, whereas the L^1 norm detected it only on July 1, 2024.

Overall, these findings suggest that spikes in the Wasserstein Distance and the L^1 and L^2 norms of persistence landscapes correspond to periods during which actual market crises or significant episodes of instability occurred. Therefore, these metrics may serve as indicators of ongoing structural instability in small-scale financial markets, as well as tools for assessing both the magnitude and duration of structural changes. However, these metrics should be applied simultaneously, as the Wasserstein Distance exhibited better detection of the duration of a crisis and how long it affected the underlying structure of the market, while the L^1 and L^2 norms were able to detect market crises faster. It is therefore recommended to use both the L^1 and L^2 norms, since, as observed in the 2024 case, one of them may produce a crisis signal earlier than the other.

METRICS ANALYSIS

The results of testing and comparing the H_0 and H_1 homologies reveal that H_1 produces false signals, which makes it unsuitable as the primary measure for detecting market crises in the Baltic market.

Specifically, across all tested window sizes, the Wasserstein-distance graphs show threshold exceedances during periods that do not correspond to known major market crises or events that could have caused a significant change in market structure (Figures 4-6 and Tables A12-A24). For instance, Table A15, which presents the threshold-exceeding Wasserstein-distance values for H_1 homology using a 20-day sliding window, shows threshold exceedances in 2016, 2017, 2018, 2019, 2022 and 2023. Most of these exceedances were short-lived and lasted only one or two trading days, and since they occurred during periods with no known major market crises, they can be considered noise. Similar signals occurring during periods of no major market crisis can be observed in Tables A16, A19, A20, A23, and A24. This

behaviour was observed across all three tested window sizes, so increasing the window size does not reduce the noise when using H_1 homology. It is also important to note that increasing the threshold value removes some of the known major crises, as seen in Tables A13, A17, and A18, where only the 2022 market crisis was detected. Similarly, Table A22 shows only a peak in 2023, and Table A21 produces no threshold-exceeding periods. Table A14, however, still produces false signals in 2016 and 2018, despite the higher threshold.

The analysis of persistence landscapes and their norms obtained from H_1 Persistence diagrams revealed that they produce small values for both the L^1 and L^2 norms, as seen in the Tables B4 and B5. Moreover, the periods in which the threshold is exceeded are short-lived and occur during periods without any known major market crises, particularly in 2016, 2018, and 2019.
Taking this into account, this study primarily focuses on H_0 homology, excluding H_1 homology.

Since there is no standard and unified window size for market-crisis detection, and the choice of window size may affect the results of the study, three different window sizes - 20, 50 and 100 days - were tested and compared to determine whether one of them could be selected as the primary window size for detecting market crises in the Baltic market. Therefore, for each window size, Vietoris–Rips complexes and the corresponding persistence diagrams were constructed, and the associated Wasserstein distances, persistence landscapes and their L^1 and L^2 norms were then calculated and plotted (see Appendices A and B).

20-day window size:

Analysis of the performance of the selected 20-day sliding window for Wasserstein distance (Appendix A, Figure A1 and Tables A1-A4) revealed that high threshold values ($\mu_W + 4\sigma$ and $\mu_W + 3\sigma$) fail to capture all major market crises. At a threshold of $\mu_W + 4\sigma$ (Table A1), only the 2020 market crisis was detected, and at a threshold of $\mu_W + 3\sigma$ (Table A2) the 2020 crisis was detected but the threshold was exceeded for only six days. A threshold of $\mu_W + 1\sigma$ (Table A4) produced a false signal in 2025. The $\mu_W + 2\sigma$ threshold, in contrast, successfully detected all major crisis periods. However, due to the small window size, the threshold exceedances lasted for very short periods and did not adequately capture the post-shock phases of the market.

50-day window size:

When analysing the Wasserstein-distance graphs using thresholds $\mu_W + 4\sigma$ and $\mu_W + 3\sigma$ (Tables A5 and A6), only the 2020 COVID-19 market crisis was detected. At a threshold of $\mu_W + 2\sigma$, both the 2020 crisis and seven days associated with the 2022 war-related market crisis were detected. However, we concluded that fluctuations exceeding these high thresholds are rare and that lower threshold values are

more appropriate for crisis detection in the Baltic market, because the analysis using thresholds of $\mu_W + 4\sigma$ and $\mu_W + 3\sigma$ detected, in most cases, only the peak of a market crisis or produced no threshold exceedances at all. In contrast, the Wasserstein distance calculated using a 50-day sliding window, a threshold of $\mu_W + 1\sigma$, and H_0 homology captured all major market crises considered in the study and provided durations that correspond to the periods during which price declines occurred. Specifically, it captured both the 2020 and 2022 market crises and the 2024 crash in the OMX Riga index, which resulted in a significant structural change in the market.

100-day window size:

The analysis of the Wasserstein-distance graphs calculated using a 100-day sliding window showed that this window size produces false peaks and fails to capture major market crises. For example, as shown in Table A12, the use of a $\mu_W + 1\sigma$ threshold produced a false signal in 2021. Increasing the threshold eliminated some of the major market crises. In Table A11, using a $\mu_W + 2\sigma$ threshold, only the 2020 market crisis was detected, while the false signal in 2021 remained. Similarly, Table A10 ($\mu_W + 3\sigma$) detected only the 2020 market crisis. The use of a $\mu_W + 4\sigma$ threshold (Table A9) produced no threshold-exceeding periods at all.

Taking everything into account, this study focuses on analysing H_0persistence diagrams constructed using 50-day sliding windows and employs a $\mu_W + 1\sigma$ threshold to detect unusual changes in market structure.

CONCLUSION

Overall, the results of this study demonstrate that Topological Data Analysis can be used as a tool for contemporaneous detection of structural changes in small-scale financial markets. Specifically, we found that persistence diagrams and persistence landscapes can capture changes in market structure during different market states, while peaks in the Wasserstein distance and L^1 and L^2 norms of persistence landscapes consistently coincided with periods of market crises, including the COVID-19 crisis in 2020, the outbreak of the war in Ukraine in 2022, and the 2024 delisting of JSC "Latvijas Gāze".

This study contributes to the existing literature by demonstrating that Topological Data Analysis can be applied not only to large, high-liquidity financial markets, as investigated in previous studies, but also to small-scale financial markets such as the Baltic market. Moreover, this approach allowed us to detect not only global financial crises but also local market events that significantly affected market structure.

Possible limitations of this study should also be considered. The threshold used in this study was selected empirically and may not be optimal for all financial markets. Comparing multiple small-scale financial markets would also help evaluate the generalizability of the proposed framework.

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APPENDIX

Appendix A

Wasserstein Distance Dynamics and Threshold Exceedance Analysis

Figure A1

Wasserstein Distance Dynamics for Zero-Dimensional Homology (H_0, Window Size = 20 Days)

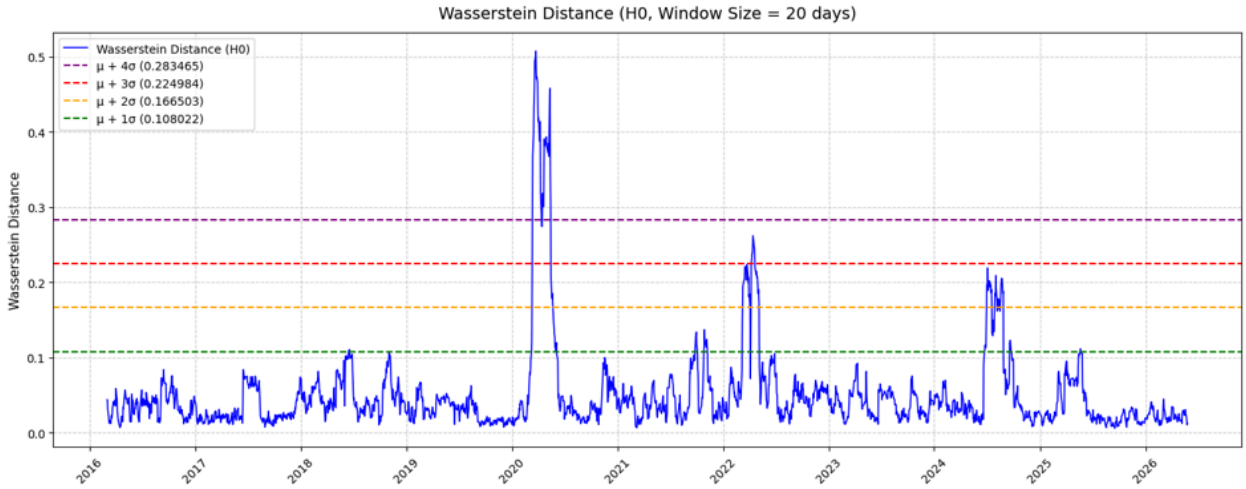

Table A1

Continuous Threshold-Exceeding Periods (μ+4σ) for H_0 (Window Size = 20 Days)

Exceeded Periods: H0, Window = 20d, Threshold μ + 4σ (0.283465)

Start date	End date	Calendar days	Trading days	Peak wasserstein
12.03.2020	09.04.2020	29	21	0.507327
15.04.2020	30.04.2020	16	12	0.393214
05.05.2020	14.05.2020	10	8	0.457831

Table A2

Continuous Threshold-Exceeding Periods (μ+3σ) for H_0 (Window Size = 20 Days)

Exceeded Periods: H0, Window = 20d, Threshold μ + 3σ (0.224984)

Start date	End date	Calendar days	Trading days	Peak wasserstein
12.03.2020	09.04.2020	29	21	0.507327
14.04.2020	30.04.2020	17	13	0.393214
05.05.2020	14.05.2020	10	8	0.457831
11.04.2022	14.04.2022	4	4	0.261896
19.04.2022	20.04.2022	2	2	0.244328

Table A3

Continuous Threshold-Exceeding Periods (μ+2σ) for H_0 (Window Size = 20 Days)

Exceeded Periods: H0, Window = 20d, Threshold μ + 2σ (0.166503)

Start date	End date	Calendar days	Trading days	Peak wasserstein
12.03.2020	09.04.2020	29	21	0.507327
14.04.2020	30.04.2020	17	13	0.393214
05.05.2020	20.05.2020	16	12	0.457831
09.03.2022	01.04.2022	24	17	0.223696
07.04.2022	14.04.2022	8	6	0.261896
19.04.2022	05.05.2022	17	12	0.244328
01.07.2024	18.07.2024	18	14	0.218926
29.07.2024	14.08.2024	17	11	0.20883
19.08.2024	27.08.2024	9	6	0.205138

Table A4

Continuous Threshold-Exceeding Periods (μ+4σ) for H_0 (Window Size = 20 Days)

Exceeded Periods: H0, Window = 20d, Threshold μ + 1σ (0.108022)

Start date	End date	Calendar days	Trading days	Peak wasserstein
15.06.2018	18.06.2018	4	2	0.110501
09.03.2020	09.04.2020	32	23	0.507327
14.04.2020	30.04.2020	17	13	0.393214
05.05.2020	03.06.2020	30	21	0.457831
27.09.2021	01.10.2021	5	5	0.13377
26.10.2021	29.10.2021	4	4	0.136994
03.11.2021	05.11.2021	3	3	0.124099
08.03.2022	14.04.2022	38	26	0.261896
19.04.2022	05.05.2022	17	12	0.244328
25.06.2024	27.08.2024	64	44	0.218926
18.09.2024	20.09.2024	3	3	0.122831
20.05.2025	21.05.2025	2	2	0.111436

Figure A2

Wasserstein Distance Dynamics for Zero-Dimensional Homology (H_0, Window Size = 50 Days)

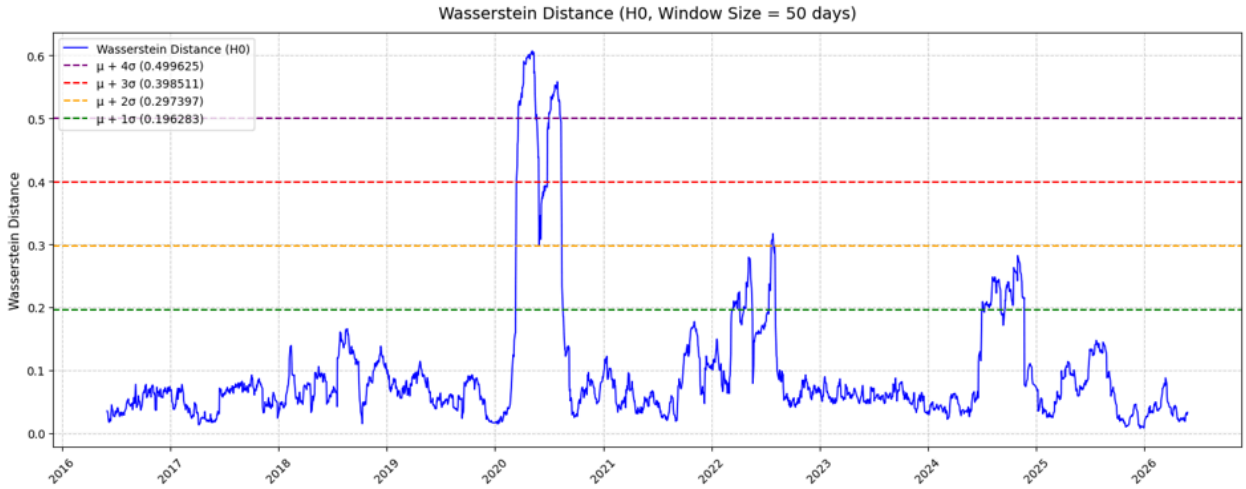

Table A5

Continuous Threshold-Exceeding Periods (μ+4σ) for H_0 (Window Size = 50 Days)

Exceeded Periods: H0, Window = 50d, Threshold μ + 4σ (0.499625)

Start date	End date	Calendar days	Trading days	Peak wasserstein
20.03.2020	09.04.2020	21	15	0.5904
14.04.2020	30.04.2020	17	13	0.601734
05.05.2020	15.05.2020	11	9	0.606679
20.05.2020	20.05.2020	1	1	0.505838
01.07.2020	07.08.2020	38	27	0.557922

Table A6

Continuous Threshold-Exceeding Periods (μ+3σ) for H_0 (Window Size = 50 Days)

Exceeded Periods: H0, Window = 50d, Threshold μ + 3σ (0.398511)

Start date	End date	Calendar days	Trading days	Peak wasserstein
13.03.2020	09.04.2020	28	20	0.5904
14.04.2020	30.04.2020	17	13	0.601734
05.05.2020	27.05.2020	23	16	0.606679
26.06.2020	11.08.2020	47	32	0.557922

Table A7

Continuous Threshold-Exceeding Periods (μ+2σ) for H_0 (Window Size = 50 Days)

Exceeded Periods: H0, Window = 50d, Threshold μ + 2σ (0.297397)

Start date	End date	Calendar days	Trading days	Peak wasserstein
12.03.2020	09.04.2020	29	21	0.5904
14.04.2020	30.04.2020	17	13	0.601734
05.05.2020	19.06.2020	46	33	0.606679
25.06.2020	13.08.2020	50	35	0.557922
22.07.2022	28.07.2022	7	5	0.316797

Table A8

Continuous Threshold-Exceeding Periods (μ+1σ) for H_0 (Window Size = 50 Days)

Exceeded Periods: H0, Window = 50d, Threshold μ + 1σ (0.196283)

Start date	End date	Calendar days	Trading days	Peak wasserstein
12.03.2020	09.04.2020	29	21	0.5904
14.04.2020	30.04.2020	17	13	0.601734
05.05.2020	19.06.2020	46	33	0.606679
25.06.2020	14.08.2020	51	36	0.557922
14.03.2022	06.04.2022	24	18	0.224312
20.04.2022	18.05.2022	29	20	0.279337
13.07.2022	04.08.2022	23	17	0.316797
04.07.2024	05.07.2024	2	2	0.208988
11.07.2024	03.09.2024	55	37	0.248067
17.09.2024	21.11.2024	66	46	0.281791

Figure A3

Wasserstein Distance Dynamics for Zero-Dimensional Homology (H_0, Window Size = 100 Days)

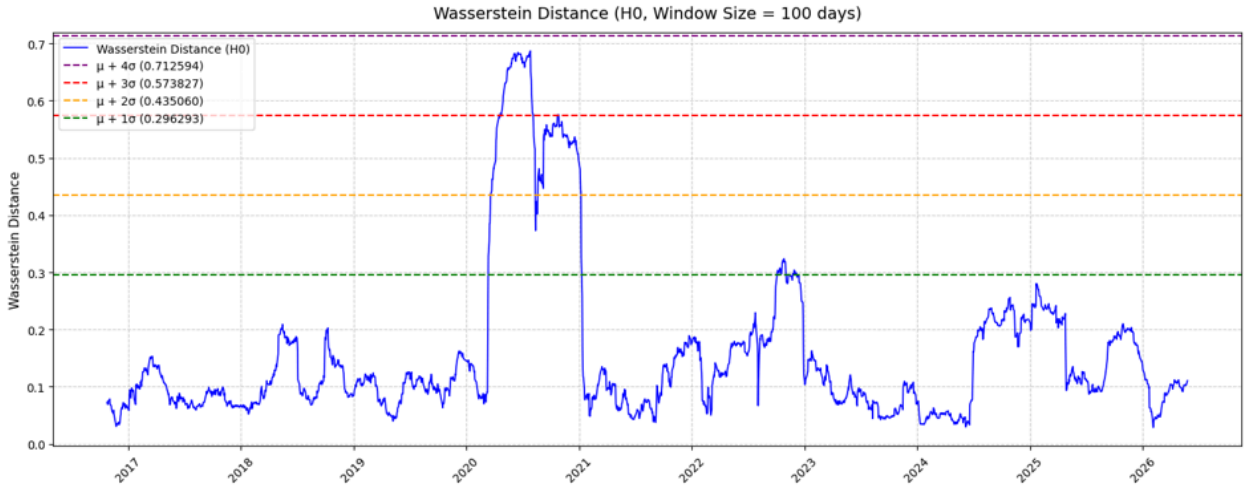

Table A9

Continuous Threshold-Exceeding Periods (μ+4σ) for H_0 (Window Size = 100 Days)

Exceeded Periods: H0, Window = 100d, Threshold μ + 4σ (0.712594)

No dates exceeded this threshold

Table A10

Continuous Threshold-Exceeding Periods (μ+3σ) for H_0 (Window Size = 100 Days)

Exceeded Periods: H0, Window = 100d, Threshold μ + 3σ (0.573827)

Start date	End date	Calendar days	Trading days	Peak wasserstein
20.04.2020	30.04.2020	11	8	0.607328
05.05.2020	19.06.2020	46	33	0.684
25.06.2020	05.08.2020	42	29	0.686711
23.10.2020	27.10.2020	5	3	0.575178

Table A11

Continuous Threshold-Exceeding Periods (μ+2σ) for H_0 (Window Size = 100 Days)

Exceeded Periods: H0, Window = 100d, Threshold μ + 2σ (0.435060)

Start date	End date	Calendar days	Trading days	Peak wasserstein
20.03.2020	09.04.2020	21	15	0.552276
14.04.2020	30.04.2020	17	13	0.607328
05.05.2020	19.06.2020	46	33	0.684
25.06.2020	11.08.2020	48	33	0.686711
21.08.2020	23.12.2020	125	87	0.575178
28.12.2020	30.12.2020	3	3	0.506398
04.01.2021	07.01.2021	4	4	0.479924

Table A12

Continuous Threshold-Exceeding Periods (μ+1σ) for H_0 (Window Size = 100 Days)

Exceeded Periods: H0, Window = 100d, Threshold μ + 1σ (0.296293)

Start date	End date	Calendar days	Trading days	Peak wasserstein
13.03.2020	09.04.2020	28	20	0.552276
14.04.2020	30.04.2020	17	13	0.607328
05.05.2020	19.06.2020	46	33	0.684
25.06.2020	23.12.2020	182	126	0.686711
28.12.2020	30.12.2020	3	3	0.506398
04.01.2021	08.01.2021	5	5	0.479924
05.10.2022	31.10.2022	27	19	0.323694
24.11.2022	06.12.2022	13	7	0.303833

Figure A4

Wasserstein Distance Dynamics for One-Dimensional Homology (H_1, Window Size = 20 Days)

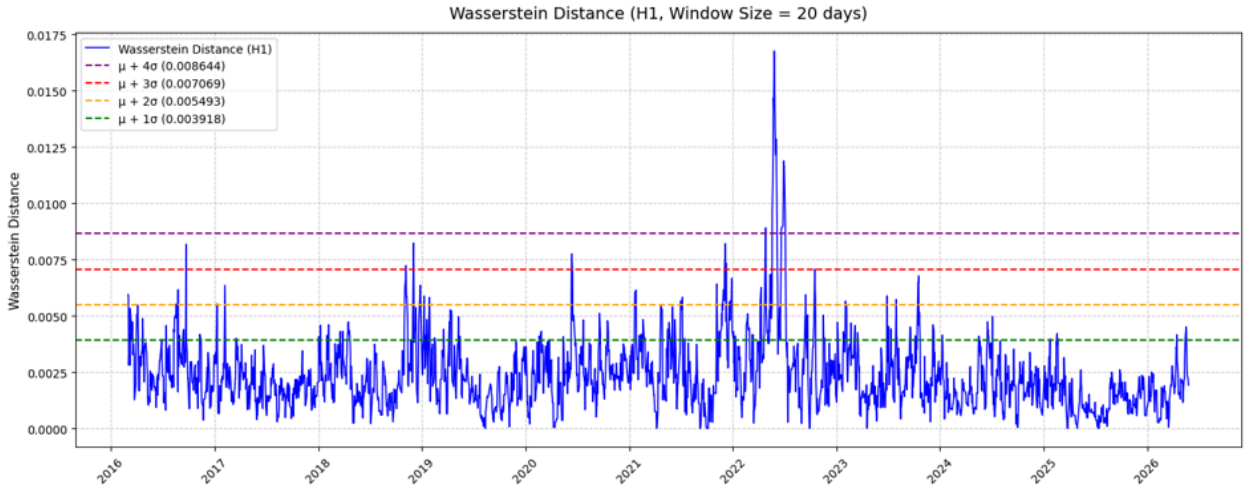

Table A13

Continuous Threshold-Exceeding Periods (μ+4σ) for H_1 (Window Size = 20 Days)

Exceeded Periods: H1, Window = 20d, Threshold μ + 4σ (0.008644)

Start date	End date	Calendar days	Trading days	Peak wasserstein
27.04.2022	27.04.2022	1	1	0.008903
20.05.2022	06.06.2022	18	11	0.016751
21.06.2022	22.06.2022	2	2	0.008898
27.06.2022	05.07.2022	9	7	0.011871

Table A14

Continuous Threshold-Exceeding Periods (μ+3σ) for H_1 (Window Size = 20 Days)

Exceeded Periods: H1, Window = 20d, Threshold μ + 3σ (0.007069)

Start date	End date	Calendar days	Trading days	Peak wasserstein
22.09.2016	22.09.2016	1	1	0.008171
06.11.2018	06.11.2018	1	1	0.007221
03.12.2018	03.12.2018	1	1	0.008223
12.06.2020	12.06.2020	1	1	0.007742
06.12.2021	09.12.2021	4	2	0.0082
25.04.2022	27.04.2022	3	3	0.008903
20.05.2022	07.06.2022	19	12	0.016751
20.06.2022	22.06.2022	3	3	0.008898
27.06.2022	05.07.2022	9	7	0.011871

Table A15

Continuous Threshold-Exceeding Periods (μ+2σ) for H_1 (Window Size = 20 Days)

Exceeded Periods: H1, Window = 20d, Threshold μ + 2σ (0.005493)

Start date	End date	Calendar days	Trading days	Peak wasserstein
02.03.2016	03.03.2016	2	2	0.005938
18.08.2016	18.08.2016	1	1	0.005518
25.08.2016	25.08.2016	1	1	0.006155
22.09.2016	22.09.2016	1	1	0.008171
09.01.2017	09.01.2017	1	1	0.005524
06.02.2017	06.02.2017	1	1	0.006346
02.11.2018	09.11.2018	8	6	0.007221
03.12.2018	04.12.2018	2	2	0.008223
27.12.2018	27.12.2018	1	1	0.006355
09.01.2019	09.01.2019	1	1	0.005869
28.01.2019	28.01.2019	1	1	0.00581
11.06.2020	15.06.2020	5	3	0.007742
21.01.2021	25.01.2021	5	3	0.00613
01.07.2021	01.07.2021	1	1	0.005656
07.07.2021	07.07.2021	1	1	0.005822
05.11.2021	05.11.2021	1	1	0.006403
25.11.2021	09.12.2021	15	9	0.0082
21.12.2021	29.12.2021	9	5	0.006659
08.04.2022	11.04.2022	4	2	0.00625
25.04.2022	28.04.2022	4	4	0.008903
12.05.2022	08.06.2022	28	16	0.016751
20.06.2022	22.06.2022	3	3	0.008898
27.06.2022	05.07.2022	9	7	0.011871
15.09.2022	15.09.2022	1	1	0.005929
17.10.2022	19.10.2022	3	3	0.007058
02.02.2023	02.02.2023	1	1	0.005645
29.06.2023	29.06.2023	1	1	0.005883
31.07.2023	31.07.2023	1	1	0.005715
16.10.2023	18.10.2023	3	3	0.006763

Table A16

Continuous Threshold-Exceeding Periods (μ+1σ) for H_1 (Window Size = 20 Days)

Exceeded Periods: H1, Window = 20d, Threshold μ + 1σ (0.003918)

Start date	End date	Calendar days	Trading days	Peak wasserstein
02.03.2016	17.03.2016	16	8	0.005938
31.03.2016	06.04.2016	7	5	0.005469
22.04.2016	22.04.2016	1	1	0.004868
28.06.2016	28.06.2016	1	1	0.003953
13.07.2016	14.07.2016	2	2	0.004563
10.08.2016	25.08.2016	16	9	0.006155
30.08.2016	30.08.2016	1	1	0.004119
12.09.2016	14.09.2016	3	3	0.004392
21.09.2016	23.09.2016	3	3	0.008171
09.11.2016	11.11.2016	3	3	0.004172
09.01.2017	10.01.2017	2	2	0.005524
06.02.2017	07.02.2017	2	2	0.006346
17.03.2017	17.03.2017	1	1	0.003997
03.01.2018	03.01.2018	1	1	0.004069
09.01.2018	09.01.2018	1	1	0.004576
31.01.2018	31.01.2018	1	1	0.004124
06.02.2018	06.02.2018	1	1	0.004597
19.03.2018	19.03.2018	1	1	0.004304
28.03.2018	28.03.2018	1	1	0.004307
16.04.2018	23.04.2018	8	6	0.004725
31.10.2018	12.11.2018	13	8	0.007221
23.11.2018	23.11.2018	1	1	0.003923
30.11.2018	12.12.2018	13	8	0.008223
21.12.2018	21.12.2018	1	1	0.004696
27.12.2018	28.12.2018	2	2	0.006355
07.01.2019	10.01.2019	4	4	0.005869
16.01.2019	18.01.2019	3	2	0.004822
25.01.2019	29.01.2019	5	3	0.00581
15.02.2019	15.02.2019	1	1	0.00402
19.03.2019	19.03.2019	1	1	0.004399
08.04.2019	16.04.2019	9	5	0.005256
10.05.2019	10.05.2019	1	1	0.004952
15.05.2019	15.05.2019	1	1	0.004084
18.02.2020	26.02.2020	9	4	0.004306
13.05.2020	14.05.2020	2	2	0.004064
11.06.2020	19.06.2020	9	7	0.007742
23.07.2020	23.07.2020	1	1	0.004809
18.09.2020	21.09.2020	4	2	0.005111
16.10.2020	19.10.2020	4	2	0.004778
19.01.2021	26.01.2021	8	6	0.00613
05.02.2021	12.02.2021	8	5	0.004178
18.02.2021	25.02.2021	8	5	0.004825
20.04.2021	27.04.2021	8	6	0.005418
17.05.2021	27.05.2021	11	6	0.004702
02.06.2021	04.06.2021	3	3	0.005395
11.06.2021	11.06.2021	1	1	0.004822
30.06.2021	08.07.2021	9	6	0.005822
15.07.2021	15.07.2021	1	1	0.003982
05.11.2021	08.11.2021	4	2	0.006403
15.11.2021	04.01.2022	51	29	0.0082
10.01.2022	10.01.2022	1	1	0.0041
18.02.2022	01.03.2022	12	7	0.005088
10.03.2022	14.03.2022	5	2	0.00415
08.04.2022	12.04.2022	5	3	0.00625
21.04.2022	03.05.2022	13	9	0.008903
10.05.2022	22.06.2022	44	30	0.016751
27.06.2022	07.07.2022	11	8	0.011871
12.09.2022	16.09.2022	5	5	0.005929
21.09.2022	22.09.2022	2	2	0.005033
13.10.2022	20.10.2022	8	6	0.007058
16.11.2022	21.11.2022	6	3	0.005016
05.01.2023	06.01.2023	2	2	0.004707
26.01.2023	27.01.2023	2	2	0.004292
01.02.2023	06.02.2023	6	4	0.005645
22.02.2023	22.02.2023	1	1	0.004671
06.03.2023	08.03.2023	3	3	0.004563
17.03.2023	17.03.2023	1	1	0.004253
29.06.2023	04.07.2023	6	3	0.005883
12.07.2023	12.07.2023	1	1	0.00442
31.07.2023	03.08.2023	4	4	0.005715
18.09.2023	20.09.2023	3	3	0.0045
06.10.2023	06.10.2023	1	1	0.004107
13.10.2023	20.10.2023	8	6	0.006763
07.12.2023	11.12.2023	5	3	0.004589
09.01.2024	10.01.2024	2	2	0.004133
17.05.2024	17.05.2024	1	1	0.00409
17.06.2024	21.06.2024	5	5	0.004733
04.07.2024	04.07.2024	1	1	0.004966
23.01.2025	23.01.2025	1	1	0.003963
17.02.2025	19.02.2025	3	2	0.004213
15.04.2026	15.04.2026	1	1	0.004151
15.05.2026	19.05.2026	5	3	0.0045

Figure A5

Wasserstein Distance Dynamics for One-Dimensional Homology (H_1, Window Size = 50 Days)

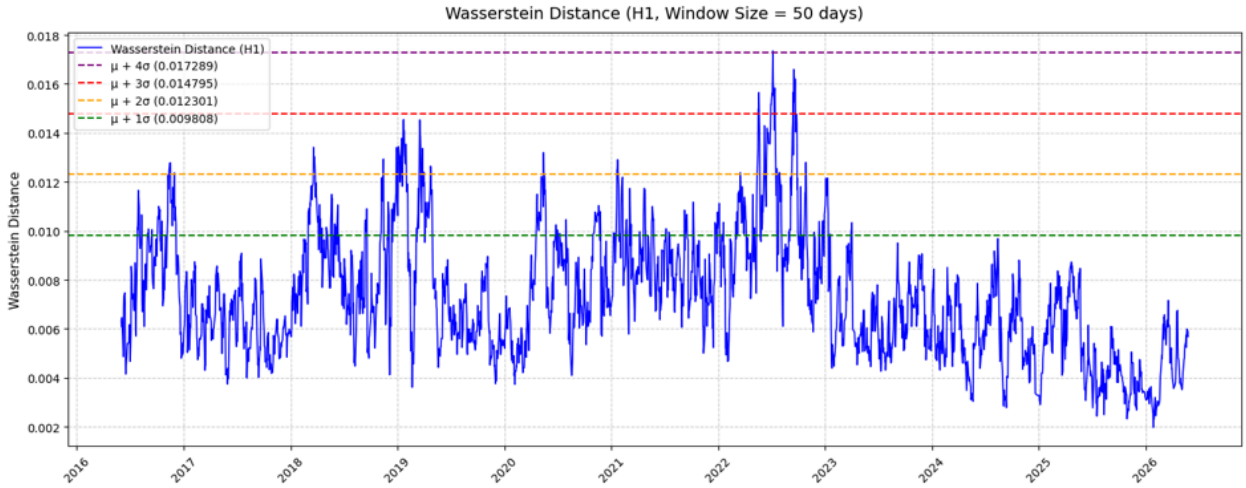

Table A17

Continuous Threshold-Exceeding Periods (μ+4σ) for H_1 (Window Size = 50 Days)

Exceeded Periods: H1, Window = 50d, Threshold μ + 4σ (0.017289)

Start date	End date	Calendar days	Trading days	Peak wasserstein
07.07.2022	07.07.2022	1	1	0.017347

Table A18

Continuous Threshold-Exceeding Periods (μ+3σ) for H_1 (Window Size = 50 Days)

Exceeded Periods: H1, Window = 50d, Threshold μ + 3σ (0.014795)

Start date	End date	Calendar days	Trading days	Peak wasserstein
18.05.2022	20.05.2022	3	3	0.015647
30.06.2022	14.07.2022	15	9	0.017347
14.09.2022	21.09.2022	8	5	0.016593

Table A19

Continuous Threshold-Exceeding Periods (μ+2σ) for H_1 (Window Size = 50 Days)

Exceeded Periods: H1, Window = 50d, Threshold μ + 2σ (0.012301)

Start date	End date	Calendar days	Trading days	Peak wasserstein
11.11.2016	16.11.2016	6	4	0.012778
29.11.2016	29.11.2016	1	1	0.012366
20.03.2018	23.03.2018	4	4	0.013409
14.11.2018	14.11.2018	1	1	0.012928
28.12.2018	28.12.2018	1	1	0.013398
04.01.2019	08.01.2019	5	3	0.013444
14.01.2019	31.01.2019	18	13	0.014543
18.03.2019	21.03.2019	4	4	0.014531
26.03.2019	01.04.2019	7	4	0.013365
24.04.2019	25.04.2019	2	2	0.01264
14.05.2020	15.05.2020	2	2	0.013195
20.01.2021	22.01.2021	3	3	0.012905
17.03.2022	18.03.2022	2	2	0.012369
16.05.2022	23.05.2022	8	6	0.015647
07.06.2022	10.06.2022	4	4	0.014292
15.06.2022	22.06.2022	8	6	0.014196
27.06.2022	15.07.2022	19	14	0.017347
20.07.2022	20.07.2022	1	1	0.012541
27.07.2022	29.07.2022	3	3	0.012382
12.09.2022	27.09.2022	16	12	0.016593
27.10.2022	28.10.2022	2	2	0.012796

Table A20

Continuous Threshold-Exceeding Periods (μ+1σ) for H_1 (Window Size = 50 Days)

Exceeded Periods: H1, Window = 50d, Threshold μ + 1σ (0.009808)

Start date	End date	Calendar days	Trading days	Peak wasserstein
26.07.2016	02.08.2016	8	6	0.011655
09.08.2016	10.08.2016	2	2	0.010654
30.08.2016	02.09.2016	4	3	0.010072
13.09.2016	14.09.2016	2	2	0.010052
03.10.2016	11.10.2016	9	7	0.010999
17.10.2016	18.10.2016	2	2	0.010397
07.11.2016	05.12.2016	29	20	0.012778
02.03.2018	29.03.2018	28	20	0.013409
03.04.2018	12.04.2018	10	7	0.011065
18.04.2018	18.04.2018	1	1	0.010114
03.05.2018	03.05.2018	1	1	0.010145
18.05.2018	23.05.2018	6	4	0.01149
29.05.2018	01.06.2018	4	3	0.010281
06.06.2018	11.06.2018	6	3	0.011055
12.09.2018	18.09.2018	7	5	0.010901
05.11.2018	14.11.2018	10	8	0.012928
30.11.2018	03.12.2018	4	2	0.011178
12.12.2018	21.12.2018	10	7	0.011125
27.12.2018	28.12.2018	2	2	0.013398
02.01.2019	05.02.2019	35	25	0.014543
07.03.2019	07.03.2019	1	1	0.010111
12.03.2019	18.04.2019	38	27	0.014531
23.04.2019	30.04.2019	8	6	0.01264
20.04.2020	30.04.2020	11	9	0.011496
05.05.2020	22.05.2020	18	13	0.013195
25.06.2020	30.06.2020	6	3	0.010244
08.07.2020	08.07.2020	1	1	0.009897
30.07.2020	30.07.2020	1	1	0.010456
30.10.2020	25.11.2020	27	16	0.011034
08.01.2021	12.01.2021	5	2	0.009914
18.01.2021	26.01.2021	9	7	0.012905
01.02.2021	10.02.2021	10	7	0.012191
19.02.2021	22.02.2021	4	2	0.010463
03.03.2021	09.03.2021	7	4	0.011731
07.04.2021	08.04.2021	2	2	0.009963
21.04.2021	28.04.2021	8	6	0.011735
19.05.2021	25.05.2021	7	5	0.010992
07.07.2021	07.07.2021	1	1	0.010086
16.07.2021	19.07.2021	4	2	0.009952
08.09.2021	08.09.2021	1	1	0.010844
13.09.2021	16.09.2021	4	4	0.01067
07.10.2021	08.10.2021	2	2	0.011172
21.12.2021	23.12.2021	3	3	0.010335
29.12.2021	05.01.2022	8	5	0.011119
13.01.2022	13.01.2022	1	1	0.009851
19.01.2022	19.01.2022	1	1	0.010344
02.03.2022	02.03.2022	1	1	0.010373
07.03.2022	07.04.2022	32	23	0.012369
29.04.2022	05.05.2022	7	3	0.011485
12.05.2022	24.05.2022	13	9	0.015647
30.05.2022	22.06.2022	24	18	0.014292
27.06.2022	20.07.2022	24	17	0.017347
27.07.2022	05.08.2022	10	8	0.012382
31.08.2022	30.09.2022	31	22	0.016593
10.10.2022	11.10.2022	2	2	0.011803
24.10.2022	28.10.2022	5	5	0.012796
25.11.2022	25.11.2022	1	1	0.009942
12.12.2022	12.12.2022	1	1	0.010384
23.12.2022	12.01.2023	21	12	0.012158
19.01.2023	19.01.2023	1	1	0.009867
31.03.2023	03.04.2023	4	2	0.010334

Figure A6

Wasserstein Distance Dynamics for One-Dimensional Homology (H_1, Window Size = 100 Days)

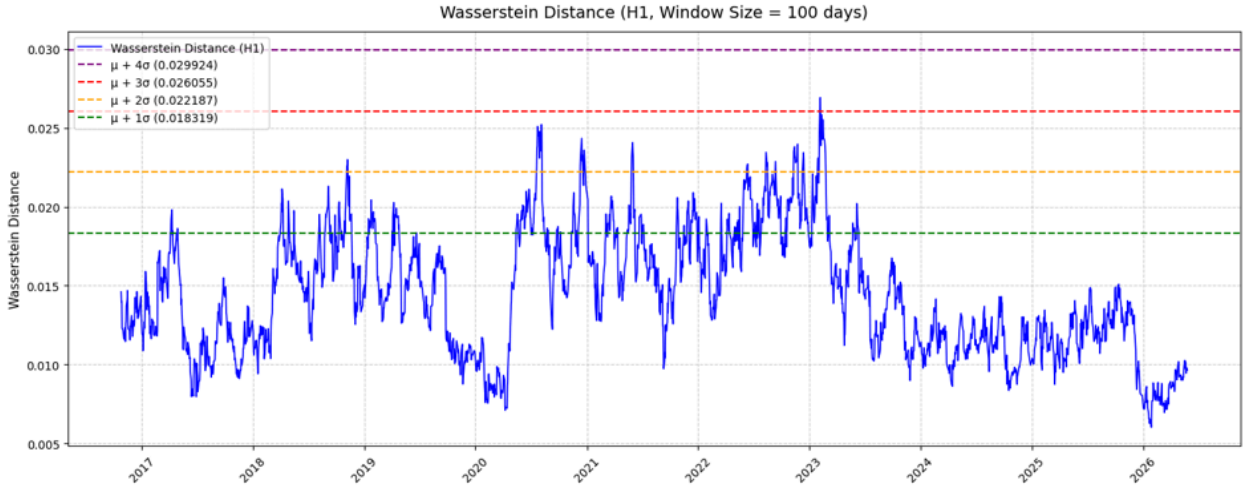

Table A21

Continuous Threshold-Exceeding Periods (μ+4σ) for H_1 (Window Size = 100 Days)

Exceeded Periods: H1, Window = 100d, Threshold μ + 4σ (0.029924)

No dates exceeded this threshold

Table A22

Continuous Threshold-Exceeding Periods (μ+3σ) for H_1 (Window Size = 100 Days)

Exceeded Periods: H1, Window = 100d, Threshold μ + 3σ (0.026055)

Start date	End date	Calendar days	Trading days	Peak wasserstein
06.02.2023	06.02.2023	1	1	0.026906

Table A23

Continuous Threshold-Exceeding Periods (μ+2σ) for H_1 (Window Size = 100 Days)

Exceeded Periods: H1, Window = 100d, Threshold μ + 2σ (0.022187)

Start date	End date	Calendar days	Trading days	Peak wasserstein
05.11.2018	08.11.2018	4	4	0.022986
20.07.2020	06.08.2020	18	14	0.0252
10.12.2020	23.12.2020	14	9	0.024334
28.05.2021	02.06.2021	6	4	0.024064
10.06.2022	13.06.2022	4	2	0.022704
11.08.2022	17.08.2022	7	4	0.023445
12.09.2022	13.09.2022	2	2	0.022865
11.11.2022	24.11.2022	14	9	0.023963
12.12.2022	14.12.2022	3	3	0.023428
30.01.2023	23.02.2023	25	16	0.026906

Table A24

Continuous Threshold-Exceeding Periods (μ+1σ) for H_1 (Window Size = 100 Days)

Exceeded Periods: H1, Window = 100d, Threshold μ + 1σ (0.018319)

Start date	End date	Calendar days	Trading days	Peak wasserstein
05.04.2017	12.04.2017	8	5	0.019806
27.04.2017	28.04.2017	2	2	0.018622
03.04.2018	10.04.2018	8	6	0.021126
27.04.2018	27.04.2018	1	1	0.02036
15.05.2018	16.05.2018	2	2	0.019748
06.08.2018	09.08.2018	4	4	0.01951
27.08.2018	13.09.2018	18	13	0.021296
27.09.2018	27.09.2018	1	1	0.019472
09.10.2018	19.10.2018	11	7	0.020296
29.10.2018	20.11.2018	23	15	0.022986
15.01.2019	06.02.2019	23	16	0.020426
03.04.2019	18.04.2019	16	9	0.020256
23.04.2019	25.04.2019	3	3	0.019411
21.06.2019	21.06.2019	1	1	0.018351
14.05.2020	20.05.2020	7	5	0.019542
28.05.2020	19.06.2020	23	17	0.020973
25.06.2020	14.08.2020	51	34	0.0252
24.08.2020	27.08.2020	4	2	0.018648
24.09.2020	28.09.2020	5	3	0.018745
06.10.2020	06.10.2020	1	1	0.018399
12.11.2020	27.11.2020	16	9	0.020887
04.12.2020	23.12.2020	20	14	0.024334
28.12.2020	30.12.2020	3	3	0.021805
04.01.2021	04.01.2021	1	1	0.020453
04.03.2021	30.03.2021	27	17	0.020669
08.04.2021	12.04.2021	5	3	0.018674
14.05.2021	07.06.2021	25	17	0.024064
22.06.2021	22.06.2021	1	1	0.019331
28.06.2021	07.07.2021	10	5	0.019519
22.10.2021	28.10.2021	7	5	0.020573
16.11.2021	16.11.2021	1	1	0.018815
25.11.2021	02.12.2021	8	6	0.020356
09.12.2021	29.12.2021	21	14	0.02089
03.01.2022	04.01.2022	2	2	0.019366
27.01.2022	31.01.2022	5	3	0.019217
17.03.2022	18.03.2022	2	2	0.020215
06.04.2022	14.04.2022	9	7	0.019573
02.05.2022	17.05.2022	16	8	0.019998
30.05.2022	22.06.2022	24	18	0.022704
27.06.2022	07.07.2022	11	6	0.020464
12.07.2022	21.07.2022	10	7	0.020208
26.07.2022	29.09.2022	66	47	0.023445
14.10.2022	29.12.2022	77	50	0.023963
10.01.2023	01.03.2023	51	35	0.026906
06.03.2023	08.03.2023	3	3	0.019443
24.05.2023	30.05.2023	7	5	0.018965
06.06.2023	12.06.2023	7	5	0.020197

Appendix B

Wasserstein Distance Dynamics and Threshold Exceedance AnalysisPersistence Landscapes and L^1, L^2 Norm Trajectories

Figure B1

Persistence Landscape for H_0 During Stable Market Period (01.06.2019–30.09.2019)

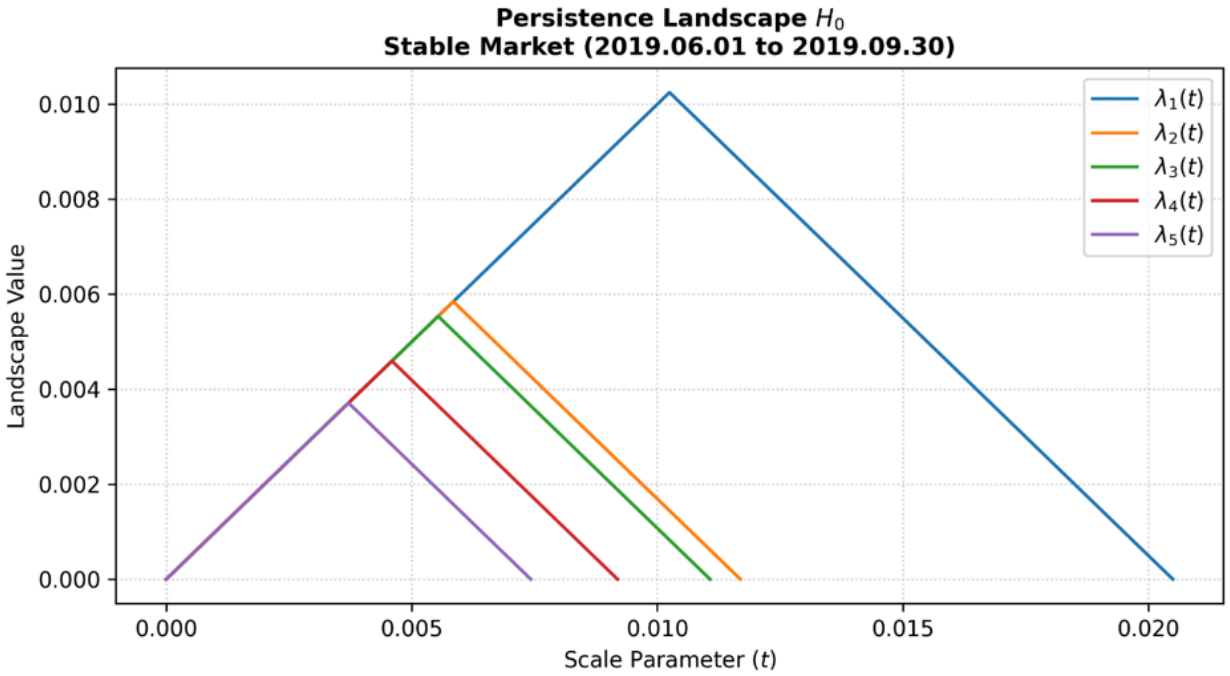

Figure B2

Persistence Landscape for H_1 During Stable Market Period (01.06.2019–30.09.2019)

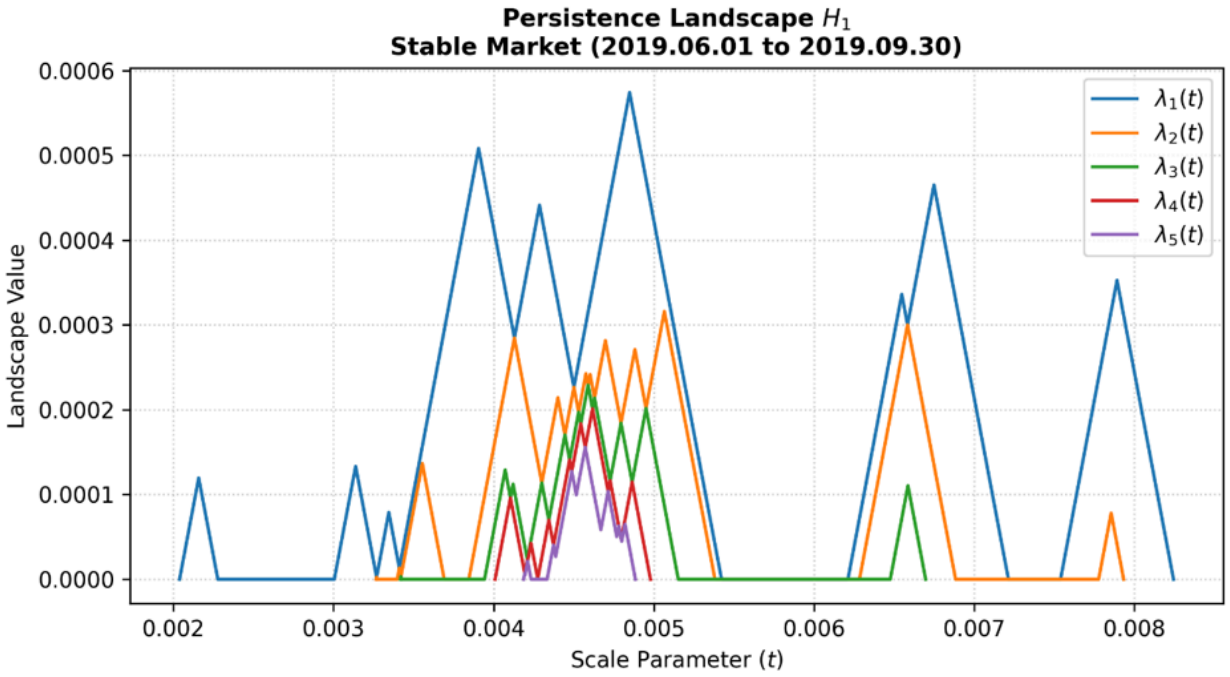

Figure B3
Persistence Landscape for H_0 During Crisis Market Period (01.02.2022–30.06.2022)

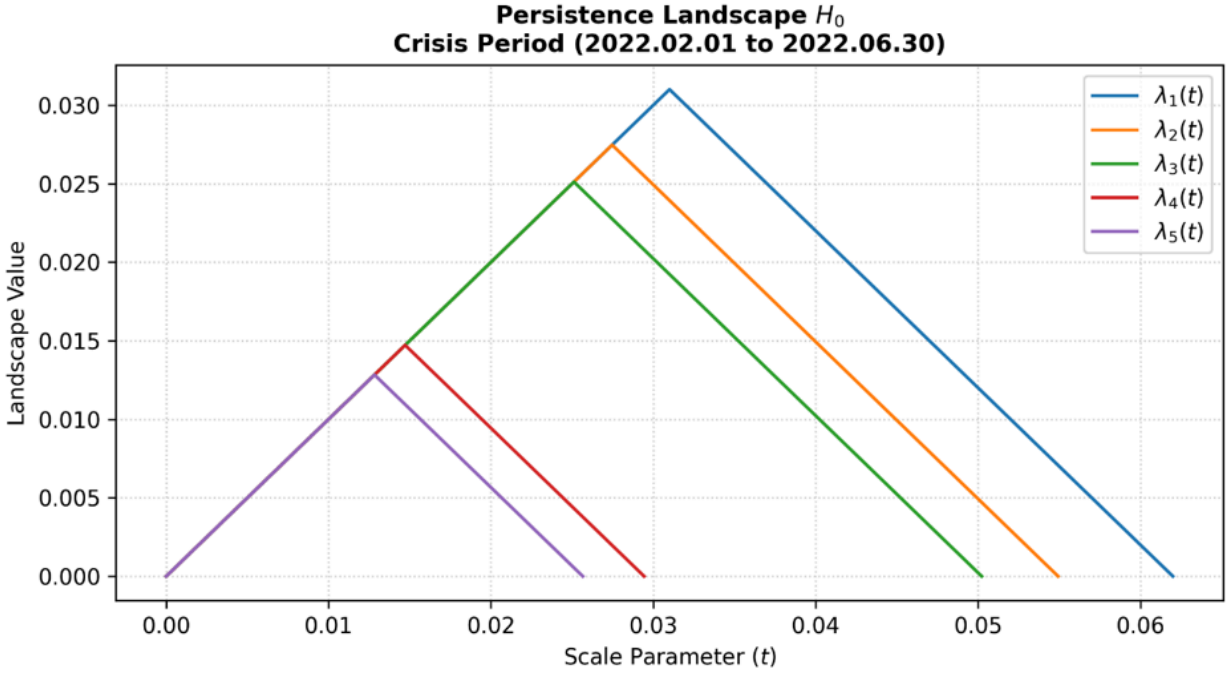

Figure B4

Persistence Landscape for H_1 During Crisis Market Period (01.02.2022–30.06.2022)

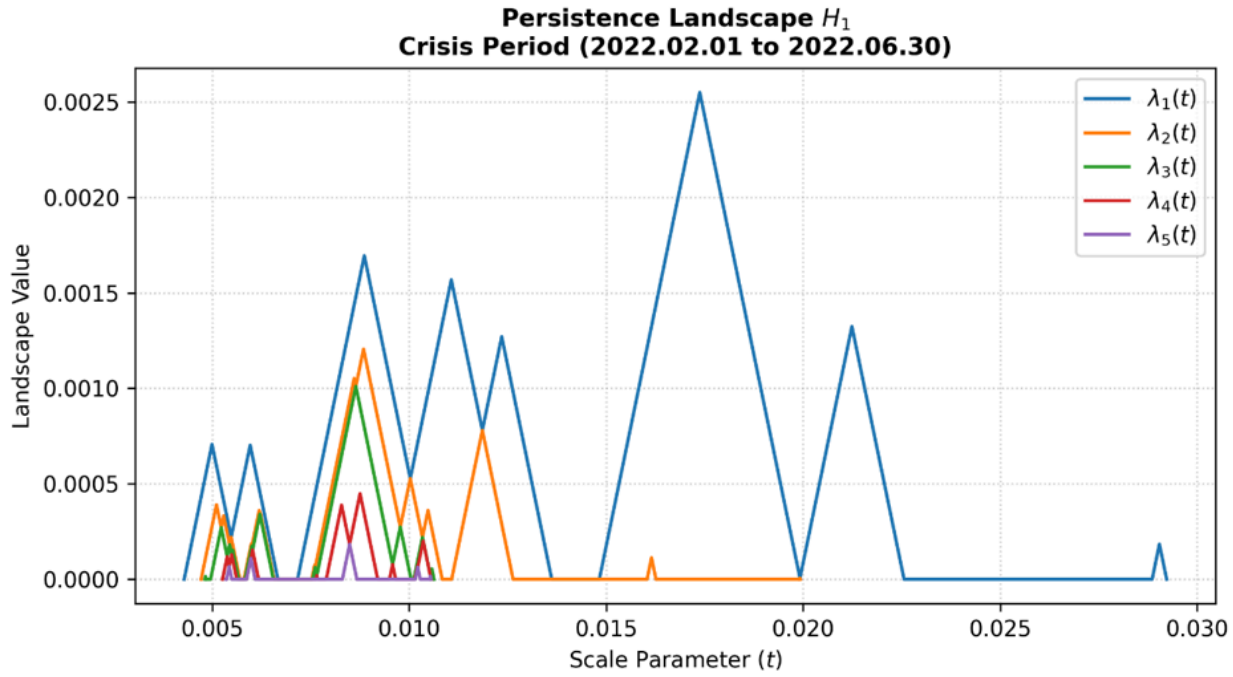

Table B1

Calculated Persistence Landscape Parameters (L^1, L^2 Norms and λmax) During Stable (01.06.2019–30.09.2019) and Crisis (01.02.2022–30.06.2022) Market Periods

Time Period	Homology	L^1 Norm	L^2 Norm	Max Value (λmax)
Stable Market (01.06.2019 – 30.09.2019)	H_0	0.000452	0.001469	0.010249
Stable Market (01.06.2019 – 30.09.2019)	H_1	0.000002	0.000025	0.000574
Crisis Period (01.02.2022 – 30.06.2022)	H_0	0.004053	0.008880	0.031001
Crisis Period (01.02.2022 – 30.06.2022)	H_1	0.000020	0.000177	0.002549

Figure B5

50-Day Rolling L^1 Norm Dynamics for H_0 With Threshold Levels (μ+1σ,μ+2σ)

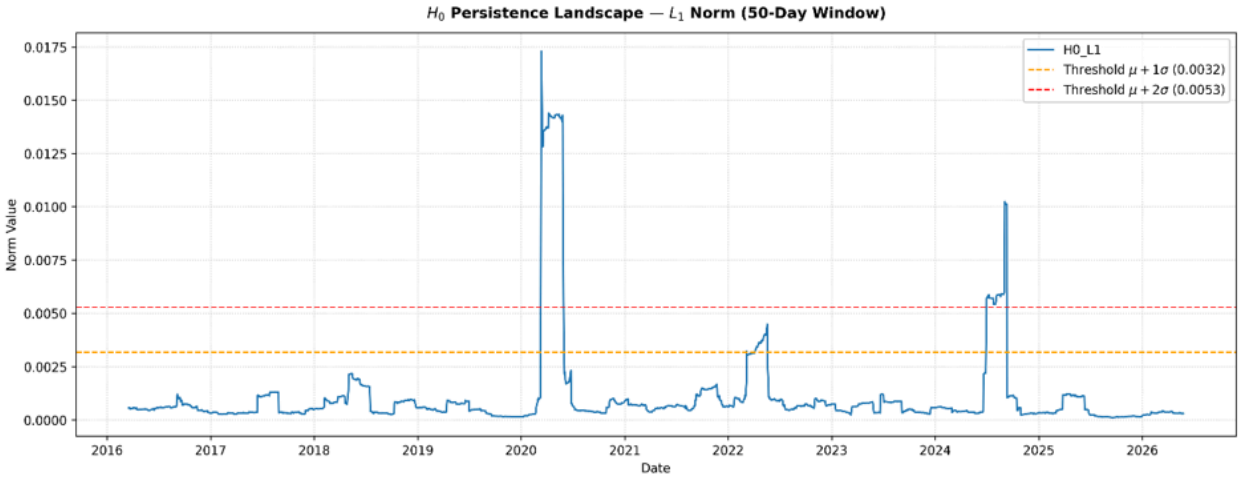

Table B2

Threshold Exceedance Intervals for H_0 L^1 Norm (50-Day Window)

`

Threshold	Start Date	End Date	Duration (Days)	Peak Value
μ+1σ (0.00317)	12.03.2020	29.05.2020	52	0.01730
	08.03.2022	08.03.2022	1	0.00323
	06.04.2022	19.05.2022	29	0.00448
	01.07.2024	10.09.2024	50	0.01024
μ+2σ (0.00529)	12.03.2020	29.05.2020	52	0.01730
	01.07.2024	10.09.2024	50	0.01024

Figure B6

50-Day Rolling L^2 Norm Dynamics for H_0 With Threshold Levels (μ+1σ,μ+2σ)

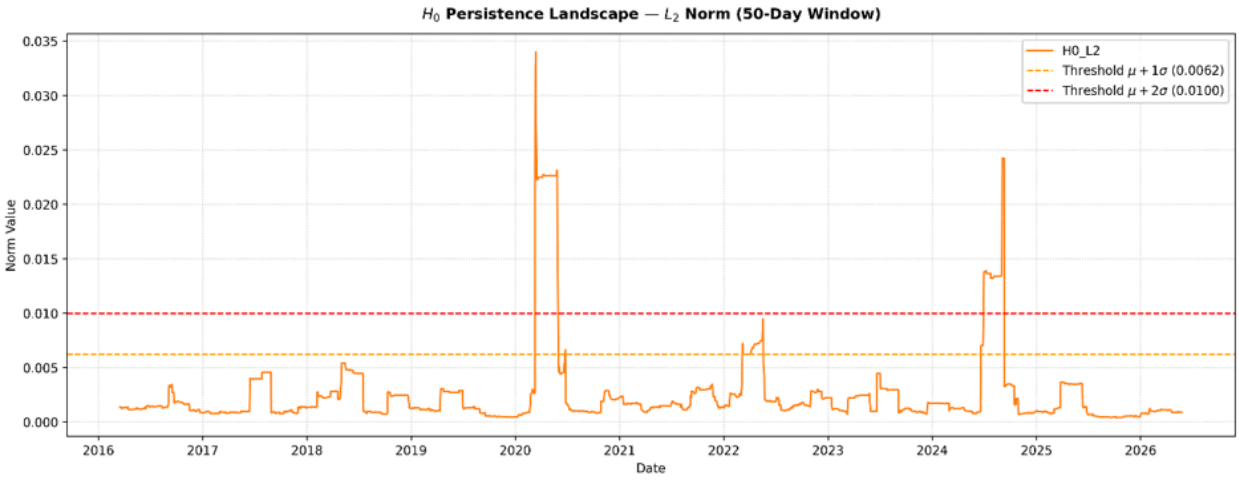

Table B3

Threshold Exceedance Intervals for H_0 L^2 Norm (50-Day Window)

Threshold	Start Date	End Date	Duration (Days)	Peak Value
μ+1σ (0.00624)	12.03.2020	29.05.2020	52	0.03399
	25.06.2020	25.06.2020	1	0.00664
	08.03.2022	08.03.2022	1	0.00720
	24.03.2022	28.03.2022	3	0.00625
	01.04.2022	19.05.2022	32	0.00946
	20.06.2024	10.09.2024	56	0.02424
μ+2σ(0.00999)	12.03.2020	29.05.2020	52	0.03399
	01.07.2024	10.09.2024	50	0.02424

Figure B7

50-Day Rolling L^1 Norm Dynamics for H_1 With Threshold Levels (μ+1σ,μ+2σ)

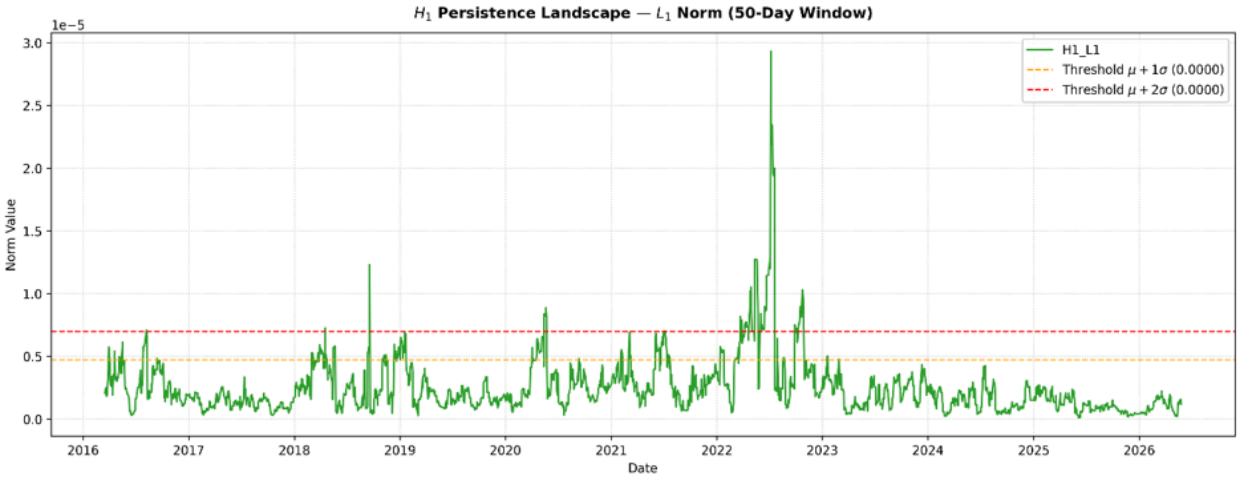

Table B4

Threshold Exceedance Intervals for H_1 L^1 Norm (50-Day Window)

Threshold	Start Date	End Date	Duration (Days)	Peak Value
μ+1σ (0.00000)	30.03.2016	01.04.2016	3	0.00001
	20.04.2016	20.04.2016	1	0.00001
	06.05.2016	06.05.2016	1	0.00000
	16.05.2016	17.05.2016	2	0.00001
	27.07.2016	10.08.2016	11	0.00001
	14.09.2016	14.09.2016	1	0.00000
	02.03.2018	08.03.2018	5	0.00001
	23.03.2018	23.03.2018	1	0.00001
	28.03.2018	10.04.2018	8	0.00001
	13.04.2018	16.04.2018	2	0.00001
	18.04.2018	18.04.2018	1	0.00001
	20.04.2018	23.04.2018	2	0.00001
	15.05.2018	23.05.2018	7	0.00001
	12.09.2018	19.09.2018	6	0.00001
	02.11.2018	07.11.2018	4	0.00001
	14.11.2018	15.11.2018	2	0.00001
	13.12.2018	20.12.2018	6	0.00001
	27.12.2018	28.12.2018	2	0.00001
	03.01.2019	22.01.2019	14	0.00001
	02.04.2020	02.04.2020	1	0.00001

	06.04.2020	06.04.2020	1	0.00000
	15.04.2020	12.05.2020	18	0.00001
	14.05.2020	22.05.2020	6	0.00001
	11.09.2020	11.09.2020	1	0.00000
	03.02.2021	08.02.2021	4	0.00001
	10.02.2021	10.02.2021	1	0.00000
	02.03.2021	09.03.2021	6	0.00001
	03.06.2021	17.06.2021	11	0.00001
	22.06.2021	13.07.2021	12	0.00001
	15.07.2021	20.07.2021	4	0.00001
	23.07.2021	23.07.2021	1	0.00000
	13.01.2022	25.01.2022	9	0.00001
	07.03.2022	16.03.2022	7	0.00001
	21.03.2022	24.05.2022	44	0.00001
	30.05.2022	20.07.2022	35	0.03000
	29.07.2022	29.07.2022	1	0.00001
	19.08.2022	22.08.2022	2	0.00000
	28.09.2022	28.10.2022	23	0.00001
	18.01.2023	19.01.2023	2	0.00001
	27.02.2023	28.02.2023	2	0.00000
$\mu+2\sigma$ (0.00001)	08.08.2016	08.08.2016	1	0.00001
	18.04.2018	18.04.2018	1	0.00001
	18.09.2018	19.09.2018	2	0.00001

	14.05.2020	22.05.2020	6	0.00001
	02.07.2021	02.07.2021	1	0.00001
	07.07.2021	07.07.2021	1	0.00001
	24.03.2022	28.03.2022	3	0.00001
	07.04.2022	20.04.2022	8	0.00001
	22.04.2022	10.05.2022	12	0.00001
	12.05.2022	23.05.2022	8	0.00001
	30.05.2022	30.05.2022	1	0.00001
	02.06.2022	20.07.2022	32	0.03000
	28.09.2022	29.09.2022	2	0.00001
	05.10.2022	05.10.2022	1	0.00001
	10.10.2022	28.10.2022	15	0.00001

Threshold	Start Date	End Date	Duration (Days)	Peak Value
$\mu + 1\sigma$ (0.00000)	30.03.2016	01.04.2016	3	0.00001
	20.04.2016	20.04.2016	1	0.00001
	06.05.2016	06.05.2016	1	0.00000
	16.05.2016	17.05.2016	2	0.00001
	27.07.2016	10.08.2016	11	0.00001
	14.09.2016	14.09.2016	1	0.00000
	02.03.2018	08.03.2018	5	0.00001
	23.03.2018	23.03.2018	1	0.00001
	28.03.2018	10.04.2018	8	0.00001

	13.04.2018	16.04.2018	2	0.00001
	18.04.2018	18.04.2018	1	0.00001
	20.04.2018	23.04.2018	2	0.00001
	15.05.2018	23.05.2018	7	0.00001
	12.09.2018	19.09.2018	6	0.00001
	02.11.2018	07.11.2018	4	0.00001
	14.11.2018	15.11.2018	2	0.00001
	13.12.2018	20.12.2018	6	0.00001
	27.12.2018	28.12.2018	2	0.00001
	03.01.2019	22.01.2019	14	0.00001
	02.04.2020	02.04.2020	1	0.00001
	06.04.2020	06.04.2020	1	0.00000
	15.04.2020	12.05.2020	18	0.00001
	14.05.2020	22.05.2020	6	0.00001
	11.09.2020	11.09.2020	1	0.00000
	03.02.2021	08.02.2021	4	0.00001
	10.02.2021	10.02.2021	1	0.00000
	02.03.2021	09.03.2021	6	0.00001
	03.06.2021	17.06.2021	11	0.00001
	22.06.2021	13.07.2021	12	0.00001
	15.07.2021	20.07.2021	4	0.00001
	23.07.2021	23.07.2021	1	0.00000
	13.01.2022	25.01.2022	9	0.00001

	07.03.2022	16.03.2022	7	0.00001
	21.03.2022	24.05.2022	44	0.00001
	30.05.2022	20.07.2022	35	0.03000
	29.07.2022	29.07.2022	1	0.00001
	19.08.2022	22.08.2022	2	0.00000
	28.09.2022	28.10.2022	23	0.00001
	18.01.2023	19.01.2023	2	0.00001
	27.02.2023	28.02.2023	2	0.00000
$\mu + 2\sigma$ (0.00001)	08.08.2016	08.08.2016	1	0.00001
	18.04.2018	18.04.2018	1	0.00001
	18.09.2018	19.09.2018	2	0.00001
	14.05.2020	22.05.2020	6	0.00001
	02.07.2021	02.07.2021	1	0.00001
	07.07.2021	07.07.2021	1	0.00001
	24.03.2022	28.03.2022	3	0.00001
	07.04.2022	20.04.2022	8	0.00001
	22.04.2022	10.05.2022	12	0.00001
	12.05.2022	23.05.2022	8	0.00001
	30.05.2022	30.05.2022	1	0.00001
	02.06.2022	20.07.2022	32	0.03000
	28.09.2022	29.09.2022	2	0.00001
	05.10.2022	05.10.2022	1	0.00001
	10.10.2022	28.10.2022	15	0.00001

Figure B8

50-Day Rolling L^2 Norm Dynamics for H_1 With Threshold Levels (μ+1σ,μ+2σ)

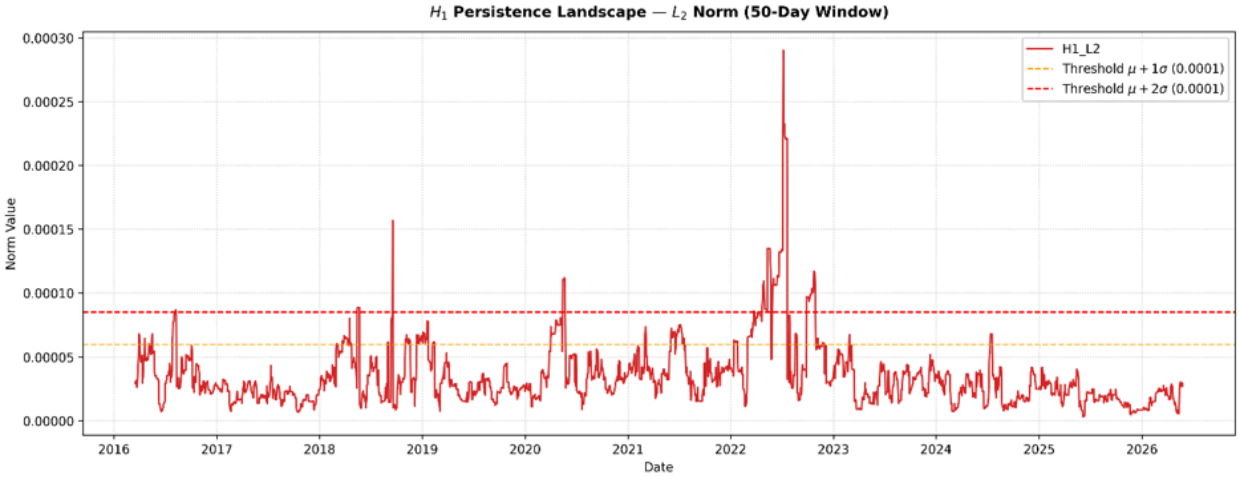

Table B5

Threshold Exceedance Intervals for H_1 L^2 Norm (50-Day Window)

Threshold	Start Date	End Date	Duration (Days)	Peak Value
μ+1σ (0.00006)	30.03.2016	01.04.2016	3	0.00007
	20.04.2016	20.04.2016	1	0.00006
	06.05.2016	06.05.2016	1	0.00006
	16.05.2016	17.05.2016	2	0.00007
	27.07.2016	10.08.2016	11	0.00009
	02.03.2018	06.03.2018	3	0.00006
	23.03.2018	11.04.2018	12	0.00007
	13.04.2018	16.04.2018	2	0.00006
	18.04.2018	23.04.2018	4	0.00008
	15.05.2018	23.05.2018	7	0.00009

	31.08.2018	03.09.2018	2	0.00006
	12.09.2018	19.09.2018	6	0.00016
	02.11.2018	12.11.2018	7	0.00007
	14.11.2018	20.11.2018	4	0.00006
	13.12.2018	22.01.2019	24	0.00008
	08.02.2019	13.02.2019	4	0.00006
	02.04.2020	12.05.2020	25	0.00008
	14.05.2020	22.05.2020	6	0.00011
	02.03.2021	05.03.2021	4	0.00007
	03.06.2021	17.06.2021	11	0.00007
	21.06.2021	13.07.2021	13	0.00008
	15.07.2021	20.07.2021	4	0.00007
	13.01.2022	25.01.2022	9	0.00006
	03.03.2022	24.05.2022	55	0.00013
	30.05.2022	20.07.2022	35	0.00029
	29.07.2022	29.07.2022	1	0.00008
	18.08.2022	24.08.2022	5	0.00007
	28.09.2022	28.10.2022	23	0.00012
	04.11.2022	07.11.2022	2	0.00006
	24.11.2022	30.11.2022	5	0.00006
	27.02.2023	28.02.2023	2	0.00007
	12.07.2024	18.07.2024	5	0.00007
$\mu+2\sigma$ (0.00008)	08.08.2016	08.08.2016	1	0.00009

	15.05.2018	23.05.2018	7	0.00009
	18.09.2018	19.09.2018	2	0.00016
	14.05.2020	22.05.2020	6	0.00011
	24.03.2022	24.03.2022	1	0.00009
	11.04.2022	13.04.2022	3	0.00009
	22.04.2022	10.05.2022	12	0.00011
	12.05.2022	24.05.2022	9	0.00013
	30.05.2022	20.07.2022	35	0.00029
	28.09.2022	28.10.2022	23	0.00012

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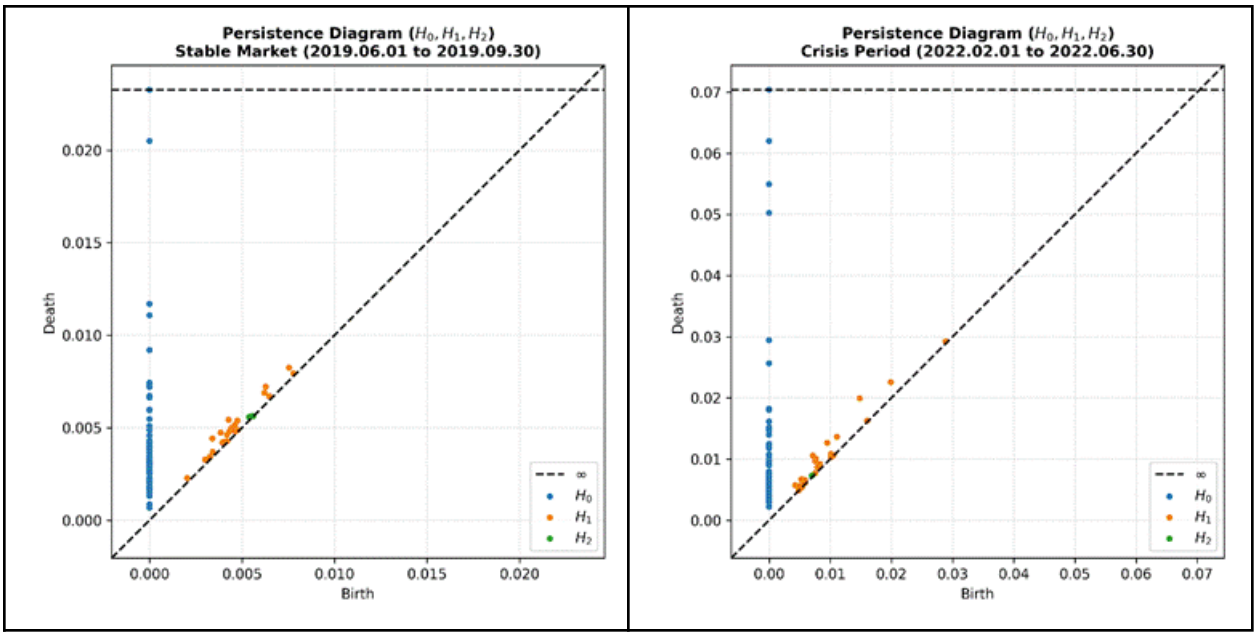
Appendix C

Persistence Diagrams Across Market Regimes

Figure C1

Persistence Diagrams (H_o, H_1, H_2) for Stable Market and Crisis Period


Persistence Diagram (H_0, H_1, H_2)
Stable Market (2019.06.01 to 2019.09.30)
Death
Birth
0.000
0.005
0.010
0.015
0.020
∞
H_0
H_1
H_2
Persistence Diagram (H_0, H_1, H_2)
Crisis Period (2022.02.01 to 2022.06.30)
Death
Birth
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
∞
H_0
H_1
H_2