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Experimental Evaluation of RC Discharge Time Constants Across a Wide Capacitance Range: Evidence for Small Non-Ideal Resistive Effects

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ABSTRACT

This study investigated how closely measured RC discharge time constants follow the theoretical relation $\tau = RC$ when capacitance is varied in a fixed-resistance circuit. Commercial capacitors from 0.068 μF to 1000 μF were discharged through an external resistor of 67.2 ± 0.1 kΩ, with three trials conducted for each capacitance. During each trial, the voltage across the capacitor was recorded using a PASCO voltage sensor, and the discharge curve was fitted to an exponential model to obtain the decay constant B; from which the experimental time constant was calculated as $\tau_{exp} = \frac{1}{B}$. Mean experimental values were compared with theoretical predictions and were analysed using linear regression and a maximum-minimum gradient method to deduce uncertainty. The measured time constants increased linearly with capacitance and closely tracked theoretical values across the range investigated. A linear fit of τ_{exp} against τ_{theory} gave y = 1.0271x + 0.0127 with $R^2 = 0.9981$, indicating strong agreement with the ideal RC model but suggesting a small systematic deviation. Interpreting this deviation with an effective-resistance model gave an inferred additional series resistance of approximately 3 ± 3 kΩ. Overall, the findings support the validity of the ideal RC approximation while showing that non-ideal resistive effects are present but small.

INTRODUCTION

Resistor-capacitor (RC) circuits describe how voltages and currents change over time in systems where a capacitor stores electrical energy and a resistor limits charge flow (York College, n.d.). In an ideal RC discharge, the potential difference across the capacitor decreases exponentially, and the characteristic timescale of this decay is the time constant τ, defined by $\tau = RC$ (Stanford University, n.d.). This concept is central to transient circuit analysis and also links directly to circuit behaviour in signal processing, since an RC network acts as a filter with a cutoff frequency $f_c = \frac{1}{2\pi RC}$ (Muthukrishnan, 2024). In this investigation, the components under study are commercial capacitors of different nominal capacitances and a fixed external resistor, allowing the time constant to be varied in a controlled way while retaining a clear theoretical relationship.

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The purpose of this investigation is to determine how closely experimentally measured RC discharge time constants agree with the theoretical prediction across a range of capacitances, and to quantify any systematic deviation that arises when real components and measurement systems are used. This topic was chosen because ideal circuit models are frequently applied in physics and engineering, yet practical designs depend on how closely real components follow those models. Accurate knowledge of τ is important in contexts such as timing circuits, sensor interfaces, and filtering, where small changes in transient response can alter performance and reliability.

The primary measurement approach uses a PASCO voltage sensor connected to a PASCO data acquisition interface to record voltage as a function of time throughout the discharge process. PASCO Capstone software is then used to display the discharge curves and apply an exponential fit to the recorded $V(t)$ data, enabling the time constant to be extracted from the full decay behaviour rather than estimated from a single threshold point. This method is appropriate because it provides consistent time sampling, reduces sensitivity to reaction time and subjective timing decisions, and supports direct comparison between experimental time constants and theoretical expectations.

RESEARCH QUESTION

To what extent does the measured RC discharge time constant, τ, *match the theoretical prediction* $\tau = RC$ *when capacitance* C *is varied in a fixed-resistance circuit?*

If non-ideal effects are present in the discharge circuit, the measured time constant will follow $\tau_{exp} \approx R_{eff} C$, where R_{eff} may differ slightly from the nominal resistor value due to summed non-idealities. Additional series contributions due to the capacitor, switch and contact resistance would increase τ through $R_{eff} \approx R + r$, while measurement loading introduces a small parallel path ($R_{eff} \approx R \| R_{probe}$) that slightly decreases τ. Therefore, τ_{exp} should remain proportional to C, and any systematic deviation from the ideal relation $\tau = RC$ can be interpreted using an effective resistance model.

THEORETICAL BACKGROUND

$$V(t) = V_0 e^{-\frac{t}{RC}} \quad (1)$$[1]

Where—

- $V(t)$ is the voltage measured at time, t in volts.
- V_0 is the initial voltage of the system, in volts

[1] Full derivation can be found in Appendix A.

- t is the time measured in seconds
- R is the external resistance of the system measured in ohms
- C is the external capacitance of the system measured in farads.
- $V_0 = \frac{Q_0}{C}$, is the initial voltage across the capacitor.

The time constant and non-ideal nature of the system

$$\tau = RC \quad (2)$$

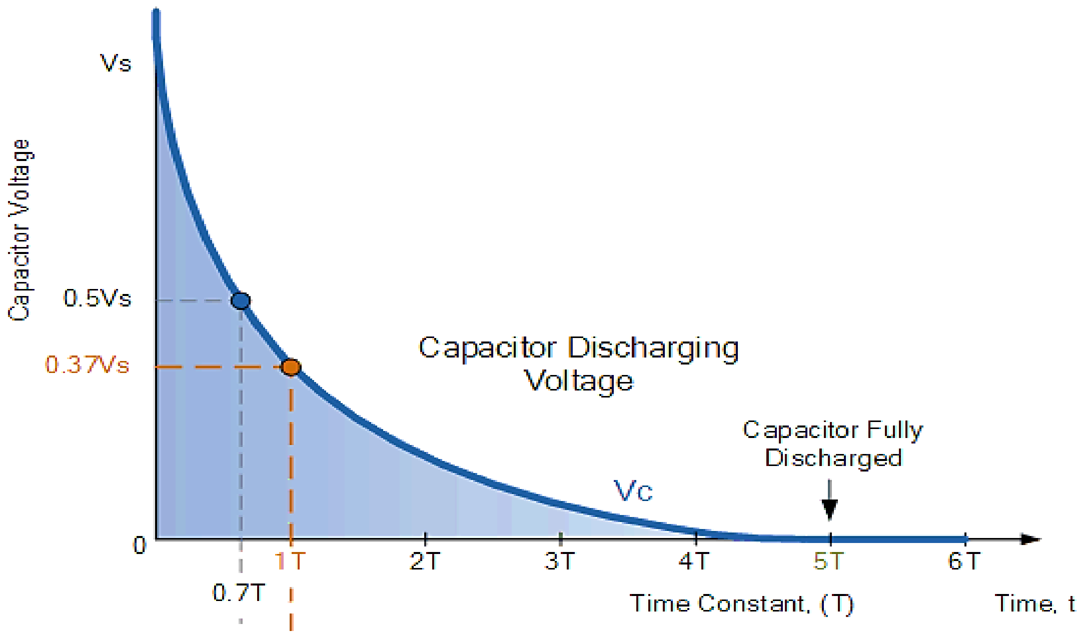
Where—

- τ is the time constant of the circuit, defined as the time taken for the voltage to decrease to e^{-1} of its initial value.

The ideal discharge model $\tau = RC$ and exponential voltage decay relies on several assumptions about the circuit and measurement conditions. First, the resistance in the discharge path is assumed to be constant and ohmic over the voltage range investigated. Second, the capacitor is treated as ideal, meaning its capacitance is constant, and energy losses due to leakage and dissipative mechanisms are negligible. Third, the measurement system is assumed not to load the circuit, so the voltage sensor input impedance is sufficiently large that it does not significantly alter the discharge current. Fourth, the power supply is assumed to be fully isolated during discharge so that no recharge path exists. Finally, stray capacitances and unintended parallel conduction paths are assumed to be small compared with the intended circuit elements. These assumptions provide a theoretical baseline; any systematic departure of τ_{exp} from τ_{theory} indicates that one or more assumptions are not fully satisfied in the experimental system.

Figure 1 (ASPENCORE, n.d.)

Exponential Discharge of Voltage in an RC Circuit

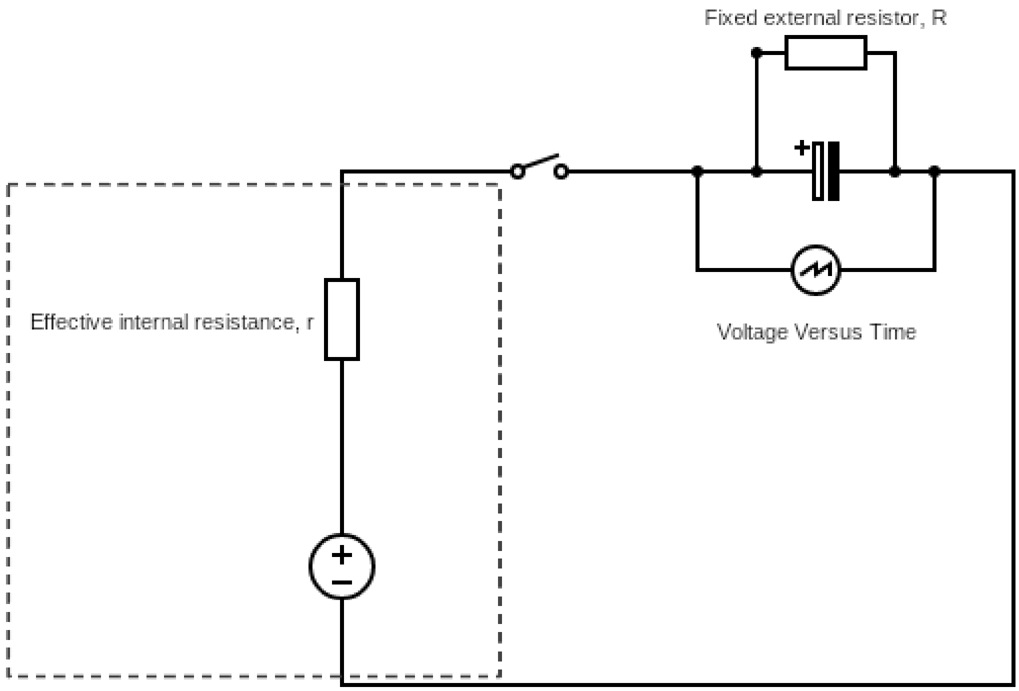


Note. Voltage across the capacitor as a function of time during capacitor discharge.

In a real experimental circuit, the resistance that governs the discharge is not necessarily equal to the nominal external resistor R. Additional series resistance can arise from multiple sources, including the internal resistance of the power supply or source circuitry (if not perfectly isolated during discharge), contact resistance at the switch and connection points, resistance of leads, and dissipative effects internal to the capacitor such as effective series resistance (ESR). For analysis, these contributions can be treated as a single summed parameter r, representing the total additional series resistance in the discharge pathway. The effective resistance can be represented as

Figure 2

Theoretical RC Discharge Circuit Including an Effective Internal Resistance r



Note. RC circuit schematic with an external resistor R, capacitor C, internal resistance r, and measurement of $V_C(t)$ across the capacitor.

Thus, mathematically, the effective resistance of the circuit can be expressed

$$R_{effective} = R + r \tag{3}$$

Where—

- r is the internal resistance of the circuit measured in ohms.

Giving an effective time constant for a non-ideal RC circuit as

$$\tau_{effective} = (R + r)C \tag{4}$$

The relation has the same formulation as the linear relationship,

$$y = mx + b \tag{5}$$

Where—

- y represents the RC time constant, τ (s), which characterises the rate of exponential voltage decay in the circuit
- x represents the capacitance of the capacitor, C(F), used in the circuit.
- m represents the gradient of the linear relationship, which corresponds to the effective resistance of the circuit, $R + r$, where R is the external resistance and r represents internal resistive contributions.
- b represents the intercept of the linear relationship (s), which is theoretically expected to be approximately zero, as a circuit with zero capacitance would have a zero-time constant.

Thus, the behaviour of a non-ideal RC circuit can be modelled using the linear relationship expressed in (4), and the deviation can be quantified through percentage error.

$$percentage\ error = \frac{\tau_{experimental} - \tau_{theoretical}}{\tau_{theoretical}} \cdot 100\% \quad (6)$$

METHODS

Table 1

Experimental Variables, Operational Handling and Justification for Validity for the RC Discharge Investigation

Variable	Variable Type	Operational method	Justification for Validity
Capacitance	Independent	Varied by substituting capacitors of different nominal capacitance values recorded in μF.	Capacitance is predicted to determine the time constant via $\tau = RC$. Varying C while holding other factors constant isolates its effect on $\tau$and allows a direct test of proportionality.
RC discharge time constant	Dependent	Determined by recording the capacitor voltage $V(t)$ during discharge using the PASCO voltage sensor and fitting to (7) in PASCO Capstone; τ calculated using (8) and (9).	Extracting τ through exponential regression uses the full discharge curve, reducing reaction-time and single-threshold error, and improving comparability across capacitances.
Fixed resistance	Controlled	The same resistor was used for all trials, determined experimentally.	Since τ is proportional to R, variation in resistance would directly alter the effect of capacitance.
Initial capacitor voltage	Controlled	Capacitor charged to a consistent voltage before each discharge; discharge run	Changes in V_0can alter signal-to-noise, fit stability, and effective behaviour if components become non-linear at higher voltages.

Variable	Variable Type	Operational method	Justification for Validity
		begins only once $V(t)$ stabilises at V_0.	
Discharge switching method and timing reference	Controlled	Same switch and wiring arrangement used. Discharge recording began only after the capacitor voltage had stabilised at the charging voltage, ensuring that each trial started from a fully charged and comparable initial condition.	Inconsistent start timing creates systematic offset and can bias exponential fits, particularly for short τ.
Data-logging sampling rate and sensor configuration	Controlled	Same PASCO voltage sensor, range, and sampling rate used for every run; identical software settings for collection duration.	Sampling rate affects how well the exponential curve is resolved. Changing it alters fit confidence and can shift fitted B.
Fit interval	Controlled	The same portion of the discharge curve used for exponential regression across trials.	Including pre-discharge points, late-time noise, or inconsistent windows can systematically change B and therefore τ.
Ambient temperature	Controlled	Trials conducted under stable room conditions; components allowed to return to ambient between runs.	Capacitor parameters especially leakage, and resistor values vary with temperature, affecting τ and deviations.
Wiring geometry and contact quality	Controlled	Lead lengths kept as short and consistent as possible; circuit layout not rearranged between trials.	Extra lead and contact resistance contribute to an effective series resistance and can appear as systematic deviation.
Capacitor polarity and pre-conditioning	Controlled	The polarity was maintained consistently and correctly.	Incorrect polarity or residual charge changes the discharge curve and can introduce drift or non-exponential behaviour, alongside potential capacitor malfunction, liquid leakage, and other safety hazards.

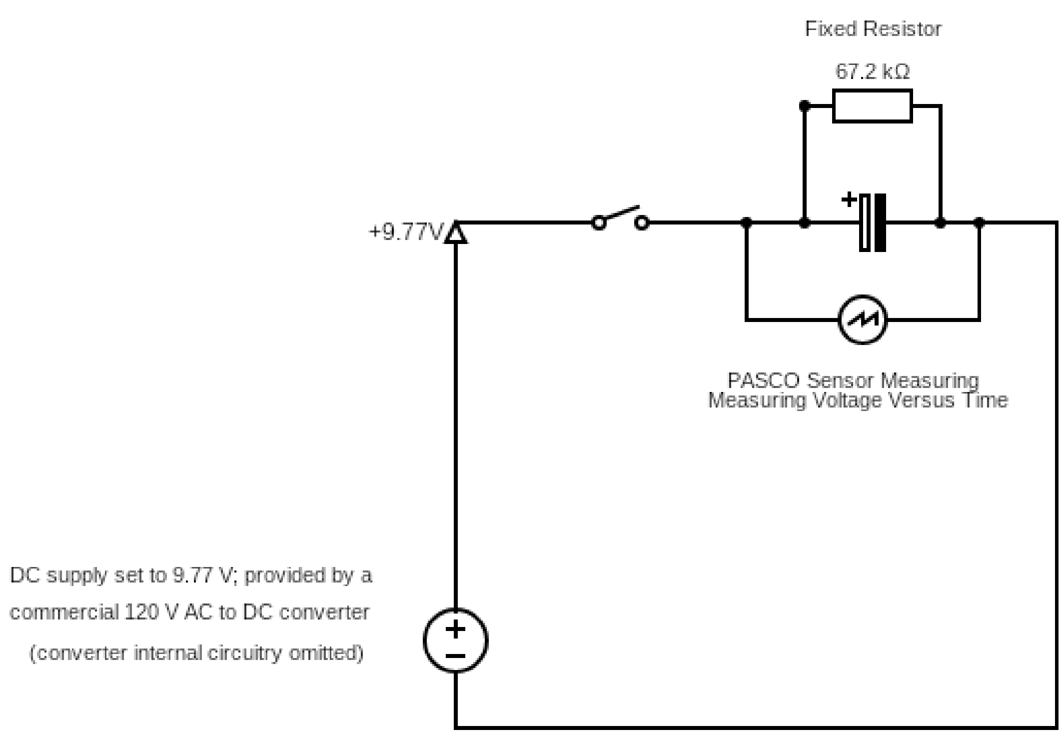
Table 2

Apparatus and Measurement Specifications, including uncertainties, for the RC Discharge Investigation

Apparatus	Key specification and Uncertainty
PASCO voltage sensor	Voltage across capacitor; resolution 5 mV.
Computer with PASCO Capstone	Fixed sampling rate; exponential regression to obtain B.
AC to DC power supply	Charging voltage (constant); ±0.01 V.
Fixed resistor	$R\ =\ 67.2 \pm 0.1\ k\Omega$.
Capacitors (multiple)	Capacitances spanning 0.068 μF to 1,000μF. Manufacturer tolerance was $\pm 20\%$.
Digital multimeter	Resistance and voltage measurements, standard digital uncertainty.
SPST toggle switch	Manual switching (charge/discharge).
Wires	Insulated copper leads.

Figure 3

Schematic Diagram of the Experimental RC Discharge Circuit Used to Measure the Time Constant τ



The investigation tested how measured RC discharge time constants deviate from the ideal model in the presence of non-ideal resistive effects. Capacitance $C$was varied as the independent variable while the external resistor $R$was held constant so that any change in τcould be attributed to C (and any additional effective series resistance), while retaining the linear prediction $\tau = RC$across several orders of magnitude. Varying $R$and Csimultaneously was avoided because it would confound attribution of deviations from theory.

A series switch controlled charging and discharge. During discharge the switch was opened to electrically isolate the power supply, ensuring the capacitor discharged only through the external resistor and preventing recharging or alternative current paths.

The experimental time constant was defined as the parameter governing the exponential decay of capacitor voltage during discharge. Voltage $V_C(t)$ was recorded continuously using a PASCO voltage sensor connected in parallel with the capacitor, and τwas extracted from exponential regression of the full V–t dataset rather than a single threshold point, reducing sensitivity to noise.

Before each run the capacitor was charged until V_0stabilised, then discharged with the supply isolated. Three trials were performed for each capacitance; $\bar{\tau}_{exp}$was taken as the mean, with trial-to-trial variation contributing to uncertainty.

Safety controls included operating below capacitor voltage ratings, maintaining correct polarity for electrolytic capacitors, and de-energising the circuit before changing components. The investigation involved low voltages and currents, had no ethical implications, and minimised environmental impact by reusing components.

The exponential-regression method was chosen over the commonly used single-time-constant method, in which the time constant, τ, is estimated as the time required for the voltage to fall to approximately 0.37 V_0 (Sedra & Smith, 2014). Although the single-threshold method is simple, it heavily depends on accurately identifying one voltage level and one corresponding time value, making it sensitive to sensor resolution, noise, sampling interval, and time-zero alignment. A logarithmic regression approach, in which $\ln(V)$ is plotted against time, could also be used (Dunford, 2010). However, this approach can give disproportionate influence to late-time data, where the voltage is small and measurement noise becomes proportionally larger. By contrast, the nonlinear exponential regression uses the full discharge curve and fits the decay constant, B, directly, reducing dependence on any single point and minimising the effect of anomalies. This makes the method more appropriate for comparing time constants across a wide capacitance range.

PROCEDURE

1. Assemble the RC circuit as shown in Figure 3, with the fixed resistor $R = 67.2\ k\Omega$ connected across the capacitor to provide the discharge path and a single-pole switch placed between the power supply and the circuit.
2. Connect the PASCO voltage sensor in parallel with the capacitor terminals and verify that the sensor polarity matches the capacitor polarity for electrolytic capacitors.
3. Set the DC power supply to a constant output voltage of approximately $9.77\ V$, ensuring this value is below the rated voltage of all capacitors used.
    a) Verify and record the initial voltage measurement and external resistor measurement using the multimeter.
4. Select the first capacitor and insert it into the circuit, ensuring correct polarity and secure connections.
5. Close the switch to charge the capacitor and wait until the measured voltage stabilises, indicating the capacitor is fully charged.
6. Begin data recording in PASCO Capstone with a fixed sampling rate and collection duration.
    a) Do not change the sample rate between trials or capacitance values.
7. Open the switch to electrically isolate the power supply and initiate capacitor discharge through the resistor only.
8. Record the voltage–time discharge data until the voltage approaches zero or the noise floor of the sensor.
9. Repeat steps 5–8 two more times for the same capacitor to obtain three discharge trials total.
10. Replace the capacitor with the next nominal capacitance value and repeat steps 4–9 for all capacitors assessed

RESULTS

Table 3[2]

Summary of Theoretical Time Constants and Mean Experimental Time Constants for Each Nominal Capacitance

C [3] / μF	ΔC / μF	τ_{theory} / s	$\Delta\tau_{theory}$ / s	$\bar{\tau}_{experimental}$ /s	$\Delta\bar{\tau}_{experimental}$ / s	Percentage Error
0.068	0.0136	0.0046	0.0009	0.004578	0.000003	0.20%
0.1	0.02	0.007	0.001	0.00723	0.00002	7.5%
0.22	0.044	0.015	0.003	0.01658	0.00003	12.1%
0.33	0.066	0.022	0.004	0.0233	0.0004	5.2%
0.47	0.094	0.032	0.006	0.0340	0.0005	7.5%
1	0.2	0.067	0.014	0.066	0.001	-1.5%
2.2	0.44	0.15	0.03	0.1758	0.0004	18.9%
3.3	0.66	0.22	0.04	0.2161	0.0004	-2.5%
4.7	0.94	0.32	0.06	0.3364	0.0003	6.5%
6.8	1.36	0.46	0.09	0.4631	0.0002	1.3%
10	2	0.7	0.1	0.7381	0.0005	9.8%
22	4.4	1.5	0.3	1.672	0.002	13.1%
33	6.6	2.2	0.4	2.459	0.005	10.9%
47	9.4	3.2	0.6	3.459	0.009	9.5%
68	13.6	4.6	0.9	4.87	0.02	6.6%
100	20	7	1	7.40	0.04	10.1%
220	44	15	3	16.7	0.2	13.1%
330	66	22	4	21.0	0.4	-5.2%
470	94	32	6	30.0	0.7	-5.0%

[2] Complete raw dataset can be found in Appendix B.
[3] Manufacture stated capacitance with a 20% percentage uncertainty.

C [3] / μF	ΔC / μF	τ_{theory} / s	$\Delta\tau_{theory}$ / s	$\bar{\tau}_{experimental}$ $/s$	$\Delta\bar{\tau}_{experimental}$ $/ s$	Percentage Error
680	136	46	9	48	2	4.2%
1000	200	70	10	70	4	3.9%

The external resistor was held constant as $67.2 \pm 0.1 k\Omega$, and the initial voltage was held constant through a power supply outputting $9.77 \pm 0.01\, V$.

The voltage–time data collected for each RC discharge were analysed using PASCO software. For each trial, the recorded voltage was fitted to an exponential decay model of the form

$$V(t) = Ae^{-Bt} + y_0 \tag{7}$$

Where—

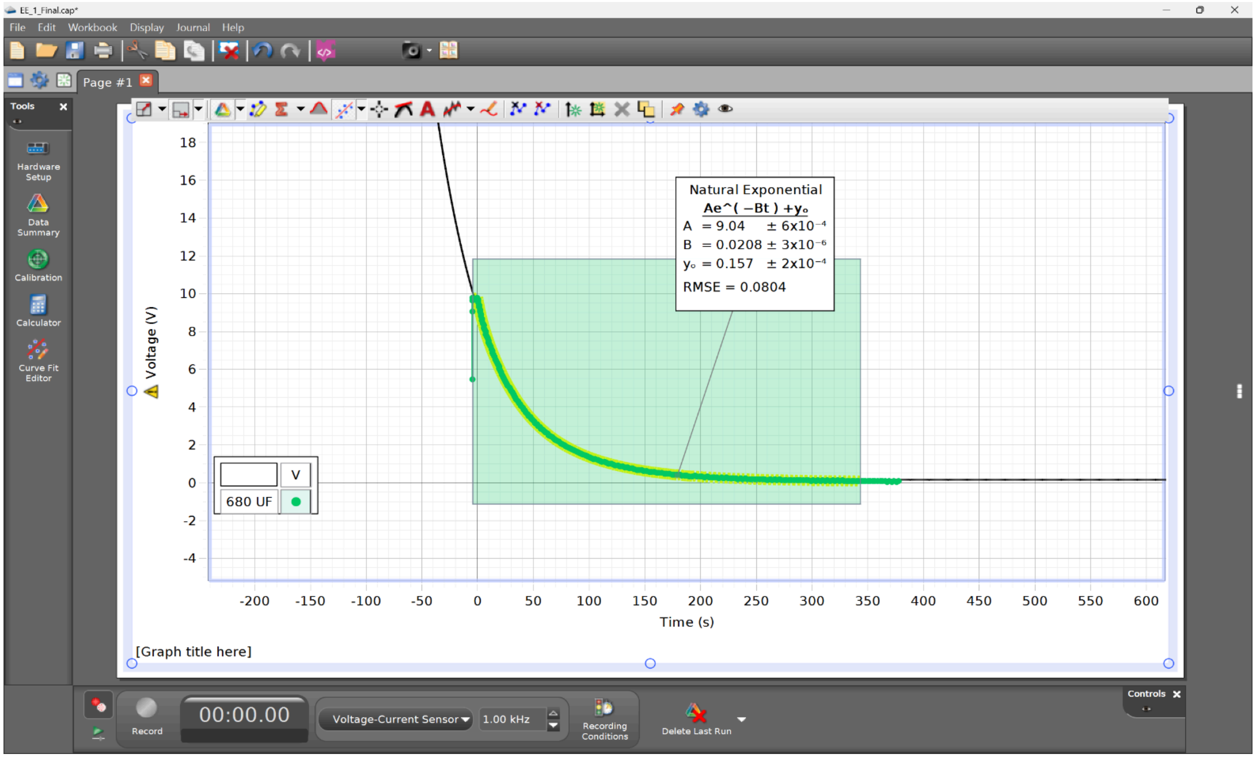
A is the initial amplitude, B is the decay constant, and y_0 is an offset parameter accounting for baseline voltage effects. The decay constant B was extracted directly from the software regression for each trial and used as the primary fitted parameter for further calculations. The experimental time constant for each trial was calculated using the relationship

$$\tau_{exp} = \frac{1}{B} \tag{8}$$

Figure 4 shows a representative discharge trace with the PASCO exponential regression used to extract B and y_0, from which $\tau_{exp} = \frac{1}{B}$ was calculated.

Figure 4

Representative capacitor discharge curve with exponential regression used to determine the decay constant B and time constant τ (Trial Number 58, Appendix B, Table B1)



The uncertainty in the experimental time constant was determined by propagating the uncertainty in the fitted decay constant according to

$$\Delta\tau_{exp} = \frac{\Delta B}{B^2} \quad (9)$$

Where—

ΔB is the uncertainty reported by PASCO for the exponential fit.

For some trials, PASCO Capstone reported extremely small regression uncertainties for B. In order to avoid reporting unrealistically small uncertainties, a lower-bound uncertainty was imposed using the voltage sensor resolution. The PASCO voltage sensor had a resolution of 5 mV, while the initial discharge voltage was approximately 9.77 V. This gives a minimum fractional voltage resolution of approximately $5{\cdot}10^{-4}$. Since, B, is obtained from the measured voltage decay, its fractional uncertainty should not be reported as smaller than the minimum resolvable fractional change in voltage. Therefore, for each trial, the uncertainty in B was taken as the larger of the PASCO-reported uncertainty and $5{\cdot}10^{-4}\,B$.

$$\Delta B = max\left(\Delta B_{pasco}, 5\cdot 10^{-4} B\right) \quad (10)^{4}$$

This ensured that the reported uncertainty reflected experimental limitations rather than numerical convergence of the regression algorithm. Three discharge trials were conducted for each capacitance value. The experimental time constant for each capacitance was taken as the mean of the three calculated trial values,

$$\bar{\tau}_{exp} = \frac{\tau_1+\tau_2+\tau_3}{3} \quad (11)$$

Instead, a conservative uncertainty was assigned as the largest of the individual trial uncertainties. The theoretical time constant for each capacitor was calculated using the ideal RC expression

$$\tau_{theory} = RC \quad (12)$$

The uncertainty in the theoretical time constant was determined by adding the relative uncertainties of the circuit components. The capacitance uncertainty was taken as ±20 percent based on the manufacturer's stated tolerance, and the resistor uncertainty was taken from the multimeter measurement as ±0.1 kΩ based on the measured value.

$$If:\ y = \frac{ab}{c}\ then:\ \frac{\Delta y}{y} = \frac{\Delta a}{a} + \frac{\Delta b}{b} + \frac{\Delta c}{c} \quad (13)$$

$$\frac{\Delta\tau_{theory}}{\tau_{theory}} = \frac{\Delta C}{C} + \frac{\Delta R}{R} = \frac{20}{100} + \frac{0.1}{67.2} \quad (14)$$

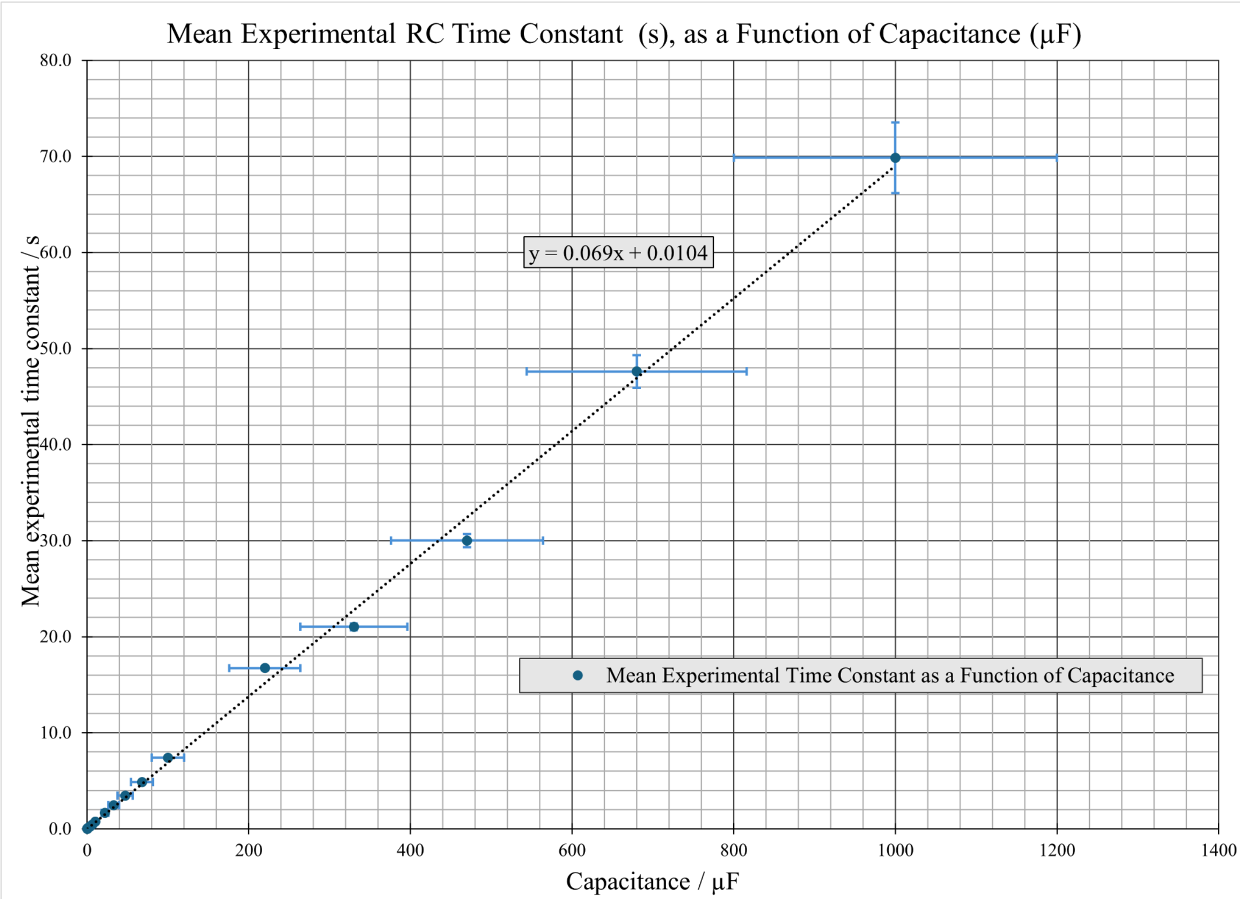
$$\frac{\Delta\tau_{theory}}{\tau_{theory}} = 20.15\% \quad (15)$$

Thus, the total relative uncertainty was 20.15 % and the absolute uncertainty in the time constant can be expressed by

$$\Delta\tau_{theory} = 0.2015\,\tau_{theory} \quad (16)$$

Figure 5

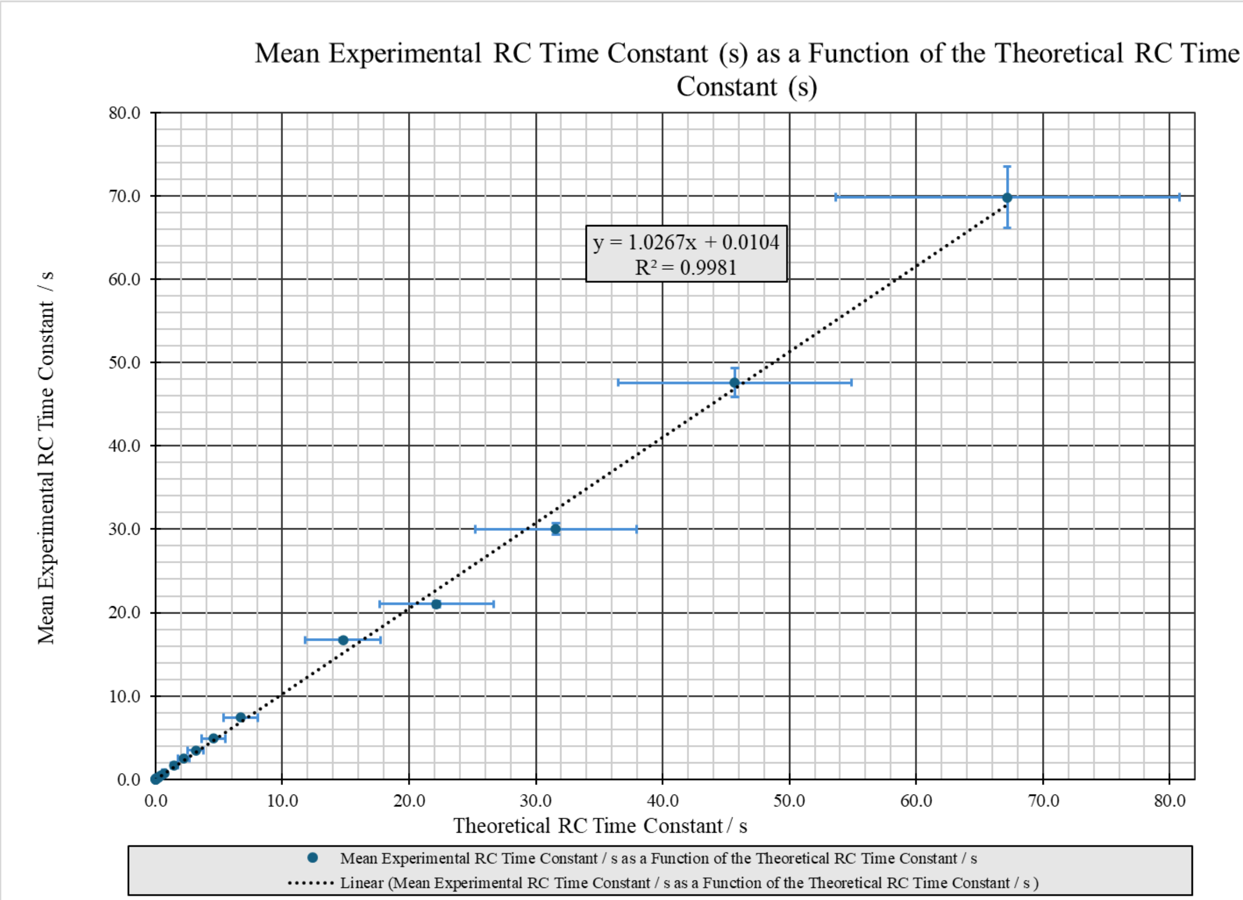
[4] Full derivation of the formula can be found in Appendix C.



Note. All error bars are included for each data point; however, some uncertainties are sufficiently small that the corresponding error bars are not visually discernible at the scale of the graph.

Figure 5 shows that the mean experimental time constant $\bar{\tau}_{exp}$ increases approximately linearly with capacitance C, consistent with first-order RC discharge behaviour where $\tau \propto C$. The best-fit relationship $\bar{\tau}_{exp} = 0.069C + 0.0127$ (with C in μF) implies a gradient of $0.069\, s\, \mu F^{-1}$. Since $1\, s\, \mu F^{-1} = 10^{6}\, s\, F^{-1}$, the fitted gradient corresponds to an effective resistance $R_{eff} \approx 0.069 \times 10^{6} \Omega = 6.9 \times 10^{4}\, \Omega$, which is close to the measured external resistance $R = 6.72 \times 10^{4} \Omega$. This supports the expectation that the dominant contribution to the discharge time constant is resistive and primarily set by the external resistor. The non-zero intercept indicates a small systematic offset that is plausibly associated with time-zero definition, switching delay, or the fitted vertical offset term (y_0) in the exponential regression, rather than a breakdown of the proportionality $\tau \propto C$. While Figure 5 establishes that $\bar{\tau}_{exp}$ scales linearly with C, Figure 6 provides a more direct test of whether the measured time constants deviate systematically from the ideal prediction $\tau_{theory} = RC$, and therefore whether an additional effective resistance is required.

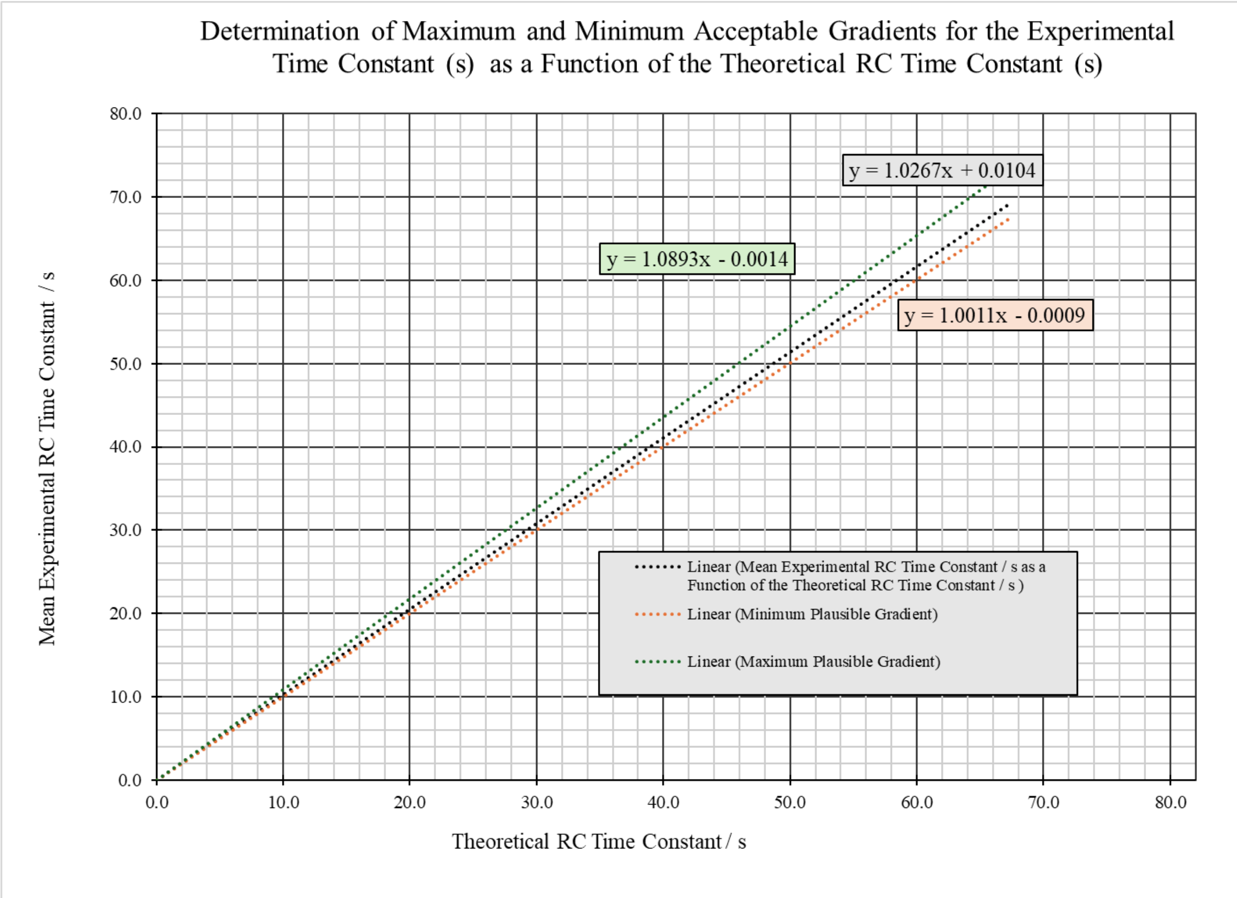
Figure 6



Note. All error bars are included for each data point; however, some uncertainties are sufficiently small that the corresponding error bars are not visually discernible at the scale of the graph.

The comparison between the mean experimental time constant and the theoretical prediction provides a consistency validation on the RC model and the assumptions used in calculating $\tau_{theoretical}$. The approximately linear relationship observed indicates that the experimental values scale proportionally with the theoretical time constants across the full range tested. The best-fit gradient of approximately 1.04 suggests that the experimental time constants are systematically slightly larger than those predicted by the ideal RC expression, which is consistent with the presence of additional series resistance not included in the theoretical calculation. The non-zero intercept mirrors the offset observed in the $\bar{\tau}_{exp}$ versus Cplot and therefore does not represent an independent deviation from the model. The increasing spread at larger time constants is consistent with the large manufacturer tolerance of the capacitors (±20%), which dominates the uncertainty in τ_{theory}. Overall, this comparison supports the conclusion that deviations between experimental and theoretical values arise primarily from non-ideal circuit elements and component tolerances rather than a failure of the RC discharge model.

Figure 7



The uncertainty in the gradient was estimated using a maximum–minimum slope method applied to the plot of mean experimental time constant, $\bar{\tau}_{exp}$, against the theoretical time constant, $\tau_{theory} = RC$. This representation is appropriate because combining $\tau_{exp} = (R + r)C$ with $\tau_{theory} = RC$ gives $\tau_{exp} = \left(\frac{R+r}{R}\right)\tau_{theory}$, so the gradient m is dimensionless and corresponds to $m = 1 + \frac{r}{R}$. The steepest and shallowest straight lines that remained consistent with the uncertainty bounds were constructed, giving $m_{min} = 1.0013$and $m_{max} = 1.0888$. The best estimate of the gradient was taken as the midpoint of these bounds, and the gradient uncertainty as half the range:

$$m = \frac{m_{max} + m_{min}}{2} \quad (17)$$

$$m = \frac{1.0888 + 1.0013}{2} = 1.04505 \quad (18)$$

$$\Delta m = \frac{m_{max} + m_{min}}{2} = \frac{1.0888 - 1.0013}{2} \quad (19)$$

$$\Delta m = 0.04375 \quad (20)$$

Rounding to reflect the uncertainty, the gradient is therefore reported as $m = 1.05 \pm 0.04$. In calculating the internal resistance accordingly, using (2) and (5)

$$\tau_{theory} = RC \Rightarrow C = \frac{\tau_{theory}}{R} \quad (21)$$

$$\tau_{exp} = (R + r)\left(\frac{\tau_{theory}}{R}\right) \quad (22)$$

$$\tau_{exp} = \left(\frac{R+r}{R}\right)\tau_{theory} \quad (23)$$

$$m = \left(\frac{R+r}{R}\right) = 1 + \frac{r}{R} \quad (24)$$

Then,

$$r = (m - 1)R. \quad (25)$$

From a fixed resistor $R = 67.2 \pm 0.1\ k\Omega$

$$r \approx (1.05 - 1)(67.2\ k\Omega) = (0.05)(67.2\ k\Omega) \quad (26)$$

$$r = 3.0274\ k\Omega \quad (27)$$

In propagating the uncertainty

$$\frac{\Delta r}{r} = \frac{\Delta(m-1)}{(m-1)} + \frac{\Delta R}{R} \quad (28)$$

$$let\ u = (m - 1) \quad (29)$$

$$\Delta u = \Delta m + \Delta 1 = \Delta m \quad (30)$$

$$\frac{\Delta r}{r} = \frac{\Delta u}{u} + \frac{\Delta R}{R} \quad (31)$$

$$\Delta r = \left(\frac{\Delta m}{m-1} + \frac{\Delta R}{R}\right)(r) \quad (32)$$

$$\Delta r \approx \left(\frac{0.04}{1.0451 - 1} + \frac{0.1}{67.2}\right)(3.0274) \quad (33)$$

$$\Delta r = 2.99 \approx 3 \quad (34)$$

$$r \approx 3 \pm 3 k\Omega \quad (35)$$

This value is small compared to the external resistance, contributing on the order of only a few percent to the total effective resistance of the circuit. This indicates that, under the conditions of this investigation, the discharge behaviour is dominated by the external resistor, with internal resistance acting as a secondary but measurable effect. The magnitude of $r$is consistent with expected contributions from the internal resistance of the voltage probe, wiring, and switching components, and does not indicate a breakdown of the RC model. As a result, while deviations from the ideal model are observable, they remain limited in scale, supporting the conclusion that the ideal RC approximation remains largely valid for this experimental configuration.

DISCUSSION

This investigation was designed around a clear theoretical model for RC discharge and used an appropriate data-processing pipeline to extract the time constant from exponential regressions. A major strength is that the method does not rely on subjective, qualitative, readings of τ; rather, $\tau_{exp} = \frac{1}{B}$ is obtained from fitted discharge curves, improving consistency across trials. A second strength is the use of a proportional comparison plot ($\bar{\tau}_{exp}$ vs $\tau_{th} = RC$), which directly tests whether experimental results scale as predicted and allows the internal resistance to be inferred through a single dimensionless factor $m = 1 + \frac{r}{R}$. The high R^2values reported for the linear models indicate that the relationship between variables is strongly linear across the tested range, supporting the validity of the RC model as the primary description of the discharge process.

The principal weakness is that the extracted internal resistance is only weakly constrained because the uncertainty is comparable to the value itself ($r \approx 3 \pm 3\ k\Omega$). This large uncertainty is driven primarily by component tolerances and how they propagate into the comparison with theory. In particular, the tolerance on the capacitance values inflates the uncertainty in $\tau_{theoretical}$, which widens the uncertainty bounds used to construct m_{min} and m_{max}, ultimately increasing Δm and therefore Δr. The investigation also uses a small number of repeats per capacitance value; which therefore limits the ability to separate random scatter from systematic effects and makes the mean $\bar{\tau}_{exp}$ more sensitive to trial-to-trial variation.

Furthermore, Systematic effects likely contributed to the non-zero intercept observed in the linear fits. A consistent positive intercept is compatible with timing offsets introduced by the definition of $t = 0$, switching delay; or the fitted offset term y_0 in the exponential regression. While this does not undermine the observed proportionality, it does indicate that the model is not capturing every practical detail of the measurement process, and it reinforces that the gradient is the only robust route for parameter extraction in this experiment. A further limitation is that the effective resistance r represents a lumped parameter that combines multiple sources (contacts, leads, switch, source behaviour, capacitor ESR and any measurement loading). As a result, even if r were better constrained, it would still not identify which physical contribution dominates without additional controls.

Several improvements would directly reduce uncertainty and strengthen the reliability of the inferred r. The highest impact change would be to measure the actual capacitance of each capacitor using an LCR meter or a calibrated capacitance function before running the discharge trials, then use these measured values in $\tau_{theory} = RC$. This would substantially reduce the horizontal uncertainty in the $\bar{\tau}_{exp}$ vs τ_{th} plot and narrow the min–max slope range therefore decreasing propagated uncertainty. Increasing the number of repeats for each capacitance value, for example, to at least four trials, would provide a more stable mean and allow a clearer distinction between random scatter and systematic offset. Procedurally, using a more consistent switching and triggering method, such as an electronic switch and a synchronised start of data logging, would reduce time-zero variability and help mitigate intercept effects. Finally, minimising contact resistance variability through fixed connectors, short leads, and repeatable wiring would reduce run-to-run changes in the effective discharge resistance.

There are also meaningful extensions that would deepen the physics without changing the core research direction. One extension would be to repeat the investigation using different capacitor types, such as comparing electrolytic to film to ceramic, to test whether the inferred r changes systematically, which would indicate a significant ESR contribution. Another extension would be to repeat the analysis for different values of the external resistor R; if r is truly an approximately constant series contribution, changing R should change the proportional deviation in a predictable way, providing a stronger test of the $m = 1 + \frac{r}{R}$ model. Together, these improvements and extensions would move the result from being consistent with "a few kΩ within large uncertainty" toward a more precise and interpretable estimate of the non-ideal resistive contributions in the discharge path.

CONCLUSION

The aim of this investigation was to determine whether the measured RC discharge time constant deviates systematically from the ideal prediction $\tau = RC$, and to use any deviation to infer an effective additional series resistance r in the discharge pathway.

This aim was addressed by (1) extracting τ_{exp} for each trial from exponential regressions of the discharge curves and (2) comparing the resulting mean values to theoretical predictions using two linear analyses. First, the plot of $\overline{\tau}_{exp}$ against capacitance C showed an approximately linear relationship, consistent with first-order RC behaviour. The best-fit gradient of $0.069\, s\, \mu F^{-1}$ corresponds to an effective resistance scale of $\sim 6.9 \times 10^4\ \Omega$, which is close to the measured external resistance $R = 6.72 \times 10^4\ \Omega$. Second, the plot of $\overline{\tau}_{exp}$ against $\tau_{theoretical} = RC$ produced a strong linear relationship (with R^2 values reported on the graphs above 0.99), indicating that $\overline{\tau}_{exp}$ is well described as proportional to τ_{theory}. Using the non-ideal model $\tau_{exp} = (R + r)C$, the gradient of this comparison corresponds to $m = 1 + \frac{r}{R}$. Applying the maximum–minimum gradient method gave $m = 1.05 \pm 0.04$, which yields

$$r = (m - 1)R \approx (3 \pm 3)\ k\Omega \qquad (36)$$

Therefore, the experimental results are consistent with a small additional effective series resistance, and they indicate that the experimental time constants are slightly larger than ideal predictions. However, because the uncertainty in r is comparable to the central value, the data only weakly constrain the magnitude of r, and the results remain consistent with the ideal case $r \approx 0$ within uncertainty.

A notable feature of the linear fits is the non-zero intercept (approximately $0.0127\ s$), which is consistent with a small systematic offset in timing definition and fitting (for example, time-zero alignment or the fitted offset term y_0), rather than a failure of the underlying proportionality. Overall, the findings align with the accepted theoretical model of RC discharge, while showing that any inferred non-ideal resistance is small relative to R and limited by the experimental uncertainty.

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APPENDIX

APPENDIX A

Derivation of The Exponential Decay Model for a Discharging Capacitor

Note. Adapted from Dunford, 2010

Consider a capacitor of capacitance C, initially charged to a potential difference V_0, discharging through an external resistor of resistance R. Applying Kirchhoff's loop law to the circuit gives:

$$V_R + V_C = 0 \quad \text{(A1)}$$

Ohm's law states that the voltage across a resistor is the current multiplied by the resistance

$$V_R = IR \quad \text{(A2)}$$

$$C = \frac{Q}{V_C} \quad \text{(A3)}$$

The initial equation can be written as

$$IR + \frac{Q}{C} = 0 \quad \text{(A4)}$$

Since current is defined as the rate of change of charge

$$I_{instantaneous} = \frac{dQ}{dt} \quad \text{(A5)}$$

Thus, the system can be described as

$$R\frac{dQ}{dt} + \frac{Q}{C} = 0 \quad \text{(A6)}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC} \quad \text{(A7)}$$

Separating variables and integrating

$$\int \frac{1}{Q} dQ = -\frac{1}{RC} \int 1 \, dt \quad \text{(A8)}$$

$$\ln |Q| = -\frac{t}{RC} + C \quad \text{(A9)}$$

where C is a constant of integration. Since Q represents the magnitude of charge stored on the capacitor, $Q > 0$, the absolute value may be omitted. Exponentiating both sides yields

$$Q(t) = e^{-\frac{t}{RC} + C} \quad \text{(A10)}$$

$$Q(t) = e^{-\frac{t}{RC}} e^{C} \quad \text{(A11)}$$

The constant e^{C} is determined by the initial condition of the system. At $t = 0$, the charge on the capacitor is $Q(0) = Q_0$, hence $e^{C} = Q_0$. The time-dependent charge is therefore

$$Q(t) = Q_0 e^{-\frac{t}{RC}} \quad \text{(A12)}$$

Since the voltage across a capacitor is given by $V = \frac{Q}{C}$, the voltage during discharge is

$$V(t) = \frac{Q(t)}{C} = V_0 e^{-\frac{t}{RC}} \quad \text{(A13)}$$

Where $V_0 = \frac{Q_0}{C}$ is the initial voltage across the capacitor.

APPENDIX B

Raw Trial Data and Calculated Time Constants for the RC Discharge Experiment

Table B1

Raw Trial Data and Calculated Time Constants for the RC Discharge Experiment With Capacitance C With Uncertainty ΔC, Fitted Decay Constant B With ΔB, Offset Voltage Y_0, and Derived $\tau_{theoretical}$ and $\tau_{experimental}$ With Respective Uncertainties

C[5] / μF	ΔC / μF	Trial Number	B / s^{-1}	ΔB / s^{-1}	Y_o / V	τ_{theory} / s	$\Delta\tau_{theory}$ / s	τ_{exp} /s	$\Delta\ \tau_{exp}\ /s$
0.068	0.0136	1	235.1	0.1	0.02	0.0046	0.0009	0.004254	0.000002
		2	168.7	0.1	0.02			0.005928	0.000004
		3	281.4	0.1	0.03			0.003554	0.000001
0.1	0.02	4	159.8	0.2	0.02	0.007	0.001	0.006258	0.000008
		5	130.4	0.3	0.04			0.00767	0.00002
		6	129	0.3	0.03			0.00775	0.00002

[5] Manufacture stated capacitance with a 20% percentage uncertainty.

C [5] / μF	ΔC / μF	Trial Number	B / s^{-1}	ΔB / s^{-1}	Y_o / V	τ_{theory} / s	$\Delta\tau_{theory}$ / s	τ_{exp} /s	$\Delta\ \tau_{exp}$ /s
0.22	0.044	7	61.02	0.08	0.03	0.015	0.003	0.01639	0.00002
		8	57.95	0.09	0.02			0.01726	0.00003
		9	62.13	0.07	0.03			0.01610	0.00002
0.33	0.066	10	42.3	0.4	0.02	0.022	0.004	0.0236	0.0002
		11	43.7	0.5	0.03			0.0229	0.0003
		12	42.6	0.5	0.02			0.0235	0.0003
0.47	0.094	13	31.4	0.3	0.02	0.032	0.006	0.0318	0.0003
		14	28.7	0.3	0.03			0.0348	0.0004
		15	28.4	0.3	0.03			0.0352	0.0004
1	0.2	16	15.2	0.1	0.01	0.067	0.014	0.0658	0.0004
		17	15.9	0.2	0.02			0.0629	0.0008
		18	14.3	0.2	0.02			0.070	0.0010
2.2	0.44	19	5.01	0.01	0.01	0.15	0.03	0.1996	0.0004
		20	6.698	0.006	0.03			0.1493	0.0001
		21	5.601	0.008	0.02			0.1785	0.0003
3.3	0.66	22	4.702	0.005	0.02	0.22	0.04	0.2127	0.0002
		23	4.398	0.005	0.04			0.2274	0.0003
		24	4.80	0.005	0.03			0.2083	0.0002
4.7	0.94	25	2.701	0.002	0.01	0.32	0.06	0.3702	0.0003
		26	3.398	0.002	0.03			0.2943	0.0002
		27	2.901	0.002	0.02			0.3447	0.0002
6.8	1.36	28	2.0003	0.0006	0.03	0.46	0.09	0.4999	0.0001
		29	2.2004	0.0007	0.02			0.4545	0.0001
		30	2.2997	0.0007	0.03			0.4348	0.0001
10	2	31	1.2002	0.0006	0.01	0.7	0.1	0.8332	0.0004
		32	1.4003	0.0006	0.02			0.7141	0.0003
		33	1.4995	0.0006	0.02			0.6669	0.0003
22	4.4	34	0.6402	0.0005	0.02	1.5	0.3	1.562	0.001
		35	0.6001	0.0005	0.02			1.666	0.001
		36	0.5597	0.0005	0.02			1.787	0.002
33	6.6	37	0.4101	0.0005	0.03	2.2	0.4	2.438	0.003
		38	0.4077	0.0005	0.03			2.453	0.003
		39	0.4022	0.0005	0.03			2.486	0.003
47	9.4	40	0.2701	0.0005	0.03	3.2	0.6	3.702	0.007
		41	0.2902	0.0005	0.05			3.446	0.006
		42	0.3097	0.0005	0.04			3.229	0.005
68	13.6	43	0.2001	0.0005	0.03	4.6	0.9	5.00	0.01

C [5] / μF	ΔC / μF	Trial Number	B / s^{-1}	ΔB / s^{-1}	Y_o / V	τ_{theory} / s	$\Delta\tau_{theory}$ / s	τ_{exp} /s	$\Delta\,\tau_{exp}$ /s
		44	0.2299	0.0005	0.06			4.350	0.009
		45	0.1901	0.0005	0.04			5.26	0.01
100	20	46	0.1532	0.0005	0.04	7	1	6.53	0.02
		47	0.1301	0.0005	0.05			7.69	0.03
		48	0.1253	0.0005	0.05			7.98	0.03
220	44	49	0.0491	0.0005	0.07	15	3	20.4	0.2
		50	0.0722	0.0005	0.05			13.85	0.09
		51	0.0627	0.0005	0.06			15.9	0.1
330	66	52	0.0741	0.0005	0.05	22	4	13.50	0.09
		53	0.0459	0.0005	0.08			22	0.2
		54	0.0360	0.0005	0.06			28	0.4
470	94	55	0.0331	0.0005	0.06	32	6	30	0.5
		56	0.0329	0.0005	0.10			30	0.5
		57	0.0340	0.0005	0.08			29	0.4
680	136	58	0.0201	0.0005	0.10	46	9	50	1
		59	0.0210	0.0005	0.12			48	1
		60	0.0220	0.0005	0.11			45	1
1000	200	61	0.0141	0.0005	0.08	70	10	71	3
		62	0.0150	0.0005	0.09			67	2
		63	0.0139	0.0005	0.09			72	3

APPENDIX C

Derivation of the Minimum Uncertainty in the Decay Constant B Based on Voltage Sensor Resolution

The voltage–time data were collected using a PASCO voltage sensor with a stated resolution of

$$\Delta V_{res} = 5mV \tag{C1}$$

During each discharge, the initial voltage across the capacitor was approximately

$$V_0 \approx 9.77V \tag{C2}$$

The smallest measurable fractional change in voltage is therefore

$$\frac{\Delta V}{V} \approx \frac{5\times10^{-3}}{9.77} \approx 5\times10^{-4} \quad \text{(C3)}$$

This quantity represents the minimum resolvable relative variation in the recorded voltage signal. The decay constant Bgoverns the rate at which the voltage decreases with time. Since $B$is determined exclusively from the temporal evolution of the measured voltage, its precision is fundamentally limited by the resolution of the voltage measurements themselves.

Hence, the fractional uncertainty in Bcannot be smaller than the fractional uncertainty in the voltage data:

$$\frac{\Delta B}{B} \geq \frac{\Delta V}{V} \quad \text{(C4)}$$

Substituting the experimental values gives

$$\frac{\Delta B}{B} \geq 5\times10^{-4} \quad \text{(C5)}$$

Rearranging yields a lower bound for the absolute uncertainty in B:

$$\Delta B_{min} = 5\times10^{-4} B \quad \text{(C6)}$$

For each trial, PASCO Capstone reports a fitted uncertainty ΔB_{PASCO}. To ensure that the quoted uncertainty reflects both statistical and instrumental limitations, the uncertainty in B was taken as

$$\Delta B = \left(\Delta B_{pasco}, 5\cdot10^{-4} B\right) \quad \text{(C7)}$$