OXFORD JOURNAL
OF STUDENT
SCHOLARSHIP

Mathematical Modeling of Violin Vibrato Using Frequency Modulation Analysis of Annotated Audio Recordings

Bill Jang
27bill.jang@gmail.com

ABSTRACT

Violin vibrato is one of the most important expressive techniques in musical performance. However, mathematical modeling and acoustical analysis may be used to quantitatively analyze it. In this study, a simple sinusoidal model was used to approximate pitch modulation in violin vibrato. Publicly available annotated violin recordings from the Music Vibrato Dataset, including the Yangjian, Laurel, CMMSD, and Coler2011 subsets, were analyzed. 199 annotated vibrato segments were yielded from a total of 17 violin recordings. Among them, 191 met the predefined inclusion criteria for the primary analysis. Pitch values were converted from Hz to cents, while removing linear pitch drift and fitting a sinusoidal model to each vibrato segment to estimate vibrato rate and amplitude. The mean vibrato rate reported 6.541 Hz, and the mean vibrato amplitude reported 16.469 cents. A mean coefficient of determination (R^2) was recorded as 0.718 by the sinusoidal model, with 90.1% of the analyzed segments producing R^2 to be greater than 0.50. Manually annotated half-cycle measurements were used for validation, demonstrating a strong positive correlation between fitted and annotated vibrato rates ($r = 0.810$, $p < 0.001$). These findings indicate that a relatively simple quantitative model can provide a moderate representation of the largely periodic structure of violin vibrato while leaving a meaningful proportion of pitch variation unexplained. In addition, this study highlights the importance of interdisciplinary connection of mathematics, physics, and music through computational analysis of artistic performance, while demonstrating how publicly available dataset may support interdisciplinary research.

Keywords: *Violin vibrato; mathematical modeling; frequency modulation; music acoustics; signal processing; pitch analysis*

INTRODUCTION

As performers can continuously manipulate pitch, dynamics, articulation, and timbre to communicate musical ideas, the violin has long been recognized as one of the most expressive musical instruments. Vibrato, a periodic oscillation of pitch around a central frequency, is regarded as one of the most significant characteristics of violin performance among aforementioned expressive techniques. Vibrato is considered as an essential component of both solo and ensemble performance as it contributes

significantly to tonal richness, musical expressiveness, and emotional communication (Seashore, 1938; Fletcher & Rossing, 1998; Rossing, Moore, & Wheeler, 2002). However, vibrato is fundamentally a measurable physical phenomenon governed by periodic changes in sound frequency even if it is traditionally researched from artistic and pedagogical perspectives. Therefore, violin vibrato is an ideal topic for interdisciplinary research that incorporates music, mathematics, and acoustical physics.

Sound comprises pressure waves determined by frequency, wavelength, amplitude, and harmonic structure from a physical perspective (Rossing et al., 2002). The performer tends to intentionally generate small, periodic fluctuations in the fundamental frequency during violin vibrato, while maintaining the overall perceived pitch of the note. Due to these fluctuations continuously occurring over time, periodic functions may be used to describe and analyze them mathematically through signal-processing techniques. Therefore, important tools for investigating musical acoustics include frequency-domain analysis, Fourier-based methods, and time-series modeling (Fletcher & Rossing, 1998; de Cheveigne & Kawahara, 2002). With these approaches, researchers can move beyond subjective description of musical expression, characterizing vibrato in the use of measurable parameters, including vibrato rate, vibrato extent, and temporal regularity.

According to prior studies, violin vibrato has been demonstrated to possess relatively stable quantitative characteristics among trained performers. For example, early psychological investigations indicated that vibrato showed systematic oscillatory behavior rather than random pitch variation (Seashore, 1938). Later, acoustical studies reported that vibrato maintained relatively consistent pitch excursions measured in musical cents while commonly representing a frequency range of approximately 5-8 Hz (Brown & Vaughn, 1996; MacLeod, 2008). According to Geringer, MacLeod, and Allen (2010), vibrato has been shown to influence perception of pitch among listeners, showing how vibrato was both a measurable acoustic property and an important perceptual cue. Putting them together, these studies report that quantitative variables may be used to effectively describe vibrato rather than purely qualitative musical terminology.

Opportunities for computational studies of musical performance have been significantly expanded by advances in digital signal processing. Researchers were able to extract high-resolution pitch contours directly from audio recordings with high accuracy thanks to the development of accurate pitch-estimation algorithms, particularly the YIN algorithm (de Cheveigne & Kawahara, 2002). It is currently possible to analyze large numbers of violin recordings with open-access annotated music datasets without specialized laboratory equipment. Therefore, the accessibility of computational music research has increased. In addition, recent studies reported how pitch, dynamics, and vibrato influenced emotional perception of violin sounds among listeners. This demonstrated that vibrato level substantially affected perceived emotional intensity and arousal (Song, Dinh, & Horner, 2024; Song, Law, Dinh, & Horner, 2025). According to comparative investigations, vibrato has been similarly shown to contribute significantly to emotional differences between bowed string instruments such as the violin and erhu (Song, Zhou, Huang, & Horner, 2024). As reported from these recent findings, violin vibrato is currently an active area of interdisciplinary research incorporating acoustics, psychology, music, and computer science.

Despite considerable progress, there are several imitations that are worth mentioning in the existing literature. Many previous investigations have primarily focused on perceptual psychology, performer training, or sophisticated acoustical experiments that required specialized equipment (Brown & Vaughn, 1996; Geringer et al., 2010). Other studies have also examined emotional responses to vibrato without generating a direct model of the physical characteristics of vibrato itself (Song et al., 2024; Song et al., 2025). Even though sinusoidal and frequency-modulation approaches are well established in the quantitative study of vibrato, thee established methods have been systematically applied by fewer studies across a large collection of annotated vibrato segments from multiple publicly available violin datasets. Therefore, this study has not contributed to the development of a novel sinusoidal model but to systematic application and validation across multiple annotated violin recording subsets to quantify vibrato rate, amplitude, and model fit.

Addressing this literature gap is valuable for both music and STEM education as quantitative modeling may be applied to an authentic artistic phenomenon. Quantitative analysis provides an opportunity to examine musical expression using reproducible computational methods rather than treating violin performance solely as an artistic practice. With integration of concepts from mathematics, acoustics, and music performance, it is feasible to illustrate how interdisciplinary approaches may deepen understanding of expressive musical techniques in the use of applied quantitative modeling.

Therefore, the purpose of this study was to analyze violin vibrato quantitatively by using a publicly available annotated dataset, while determining whether pitch modulation in violin recordings may be quantitatively characterized using an established sinusoidal mathematical model. For individual vibrato segments, vibrato rate, vibrato amplitude, and goodness-of-fit of the model were estimated, while validating fitted vibrato rates by using manually annotated half-cycle measurements. This study specifically aims to demonstrate how relatively simple quantitative models may effectively characterize one of the most important expressive features of violin performance. In particular, this study seeks to answer the research question about to what extent pitch modulation in annotated violin recordings can be represented by a sinusoidal quantitative model, and what quantitative patterns are observed in vibrato rate and vibrato amplitude. This study hypothesizes that violin vibrato will show approximately periodic pitch modulation that can be moderately represented using a sinusoidal quantitative model, with most vibrato rates falling within the commonly reported range of 5 – 8 Hz.

METHODS AND MATERIALS

Dataset

In this study, the Music Vibrato Dataset, a publicly available collection of annotated musical recordings designed for vibrato analysis, was used. The Music Vibrato Dataset was developed for ground-truth vibrato detection and parameter estimation (Yang et al., 2017). This dataset was obtained from GitHub that contained recording from several bowed-string instrument collections. For the data analysis, only the violin subsets (Yangjian, Laurel, CMMSD, and Coler, 2011) were analyzed as both annotated vibrato

regions and corresponding pitch information suitable for quantitative modeling were contained in those subsets.

A total of 17 digital audio recordings in WAV format were contained in the violin dataset together with other annotation files. These included manually labeled vibrato intervals, half-cycle vibrato annotations (Yangjian subset), and YIN-generated pitch contour files that estimates of the fundamental frequency (Hz) were contained throughout each recording. With annotated files, individual vibrato segments were isolated and analyzed without manual segmentation.

Data Processing

In all analyses, Python (Version 3.11) was used. The WAV recordings and annotation files were imported into Python using the Numpy, Pandas, and SciPy scientific computing libraries. After automatically extracting vibrato intervals identified in the annotation files, individual vibrato segments were created for the analysis.

The original pitch values were recorded in Hertz (Hz). Due to the logarithmic nature of musical pitch perception, all pitch values were converted to cents as a logarithmic unit that was commonly used in acoustics to measure pitch differences. With the conversion of data to cents, it was possible to compare pitch deviations consistently across notes with different fundamental frequencies.

A linear trend was removed from each vibrato segment before mathematical modeling to minimize long-term pitch drift related to vibrato. In addition, segments in the length shorter than 0.30 seconds or the ones containing insufficient pitch information were excluded from the primary analysis as it was reliable to estimate vibrato parameters for these recordings.

Mathematical Model

For the violin vibrato, a sinusoidal function was used representing periodic pitch modulation over time:

$$c(t) = C + \beta t + A\sin(2\pi rt+\varphi)$$

where c(t) represents pitch in cents at time t, C represents the central pitch, β represents any residual linear pitch drift, A represents the vibrato amplitude, r is the vibrato rate (Hz), and φ represents the phase angle.

For each vibrato segment, the difference between the observed pitch contour and the fitted model was minimized to estimate the sinusoidal parameters. Using the fitted sinusoidal coefficients, vibrato amplitude was calculated, and vibrato rate corresponded to the frequency producing the smallest residual error.

For Yangjin recordings, an independent estimate of vibrato rate was provided by manually annotated half-cycle measurements. With these annotations, the rates obtained from the mathematical model were validated.

Statistical Analysis

For all analyzed vibrato segments, descriptive statistics, such as the mean, standard deviation, median, interquartile range, minimum, maximum, and 95% confidence interval were calculated for vibrato rate and vibrato amplitude.

When evaluating the model, coefficient of determination (R^2), and the root mean square error (RMSE) were used. Higher R^2 values indicated that a greater proportion of the observed pitch variation was explained by the sinusoidal model, and lower RMSE values indicated smaller errors in prediction. To directly quantify the extent to which the sinusoidal model represented each vibrato segment, the proportion of unexplained pitch variance was calculated as $1 – R^2$. Therefore, an R^2 of 0.70 showed that 70% of the observed detrended pitch variation was explained by the model, while 30% remained unexplained.

The Pearson correlation coefficient was used when evaluating the agreement between manually annotated vibrato rates and model-derived vibrato rates. In addition, the accuracy of the fitted vibrato rates was quantified by using the mean absolute error (MAE), RMSE, and mean bias.

Furthermore, the relationship between vibrato rate and vibrato amplitude was examined by an exploratory Pearson correlation across all analyzed segments. Statistical summaries were generated for the complete dataset and also for each violin subset separately to assess the consistency across recordings.

RESULTS

In the initial analysis, a total of 17 violin recordings from the Music Vibrato Dataset were included. Across these recordings, a total of 199 annotated vibrato segments were identified. Among them, 198 segments produced valid sinusoidal model fits, while 191 segments were retained for the primary statistical analysis as they satisfied the predefined criteria. 8 segments were excluded from the primary analysis as they were either shorter than 0.30 seconds or failed to satisfy the criteria for reliable model fitting (Table 1).

Measure	Value
Violin recordings	17
Annotated vibrato segments	199
Valid sinusoidal fits	198
Segments included in primary analysis	191
Excluded segments	8

Table 1. Summary of the analyzed violin dataset. Summary of the violin recordings and vibrato segments included in the study after screening the dataset. The table reports the number of recordings, annotated vibrato segments, valid sinusoidal fits, and the final sample used for statistical analysis.

According to the analyzed vibrato segments, relatively consistent vibrato rates were reported across recordings. The mean vibrato rate was 6.541 Hz (SD = 0.771 Hz), with a median of 6.575 Hz. The

interquartile range was from 6.065 Hz to 7.035 Hz, and the overall range was from 4.550 to 9.530 Hz. The estimated 95% confidence interval for the mean vibrato rate was calculated to be between 6.431 and 6.651 Hz (Table 2). Among the analyzed segments, 179 out of 191 segments (93.7%) reported vibrato rates between 5 and 8 Hz (Figure 1).

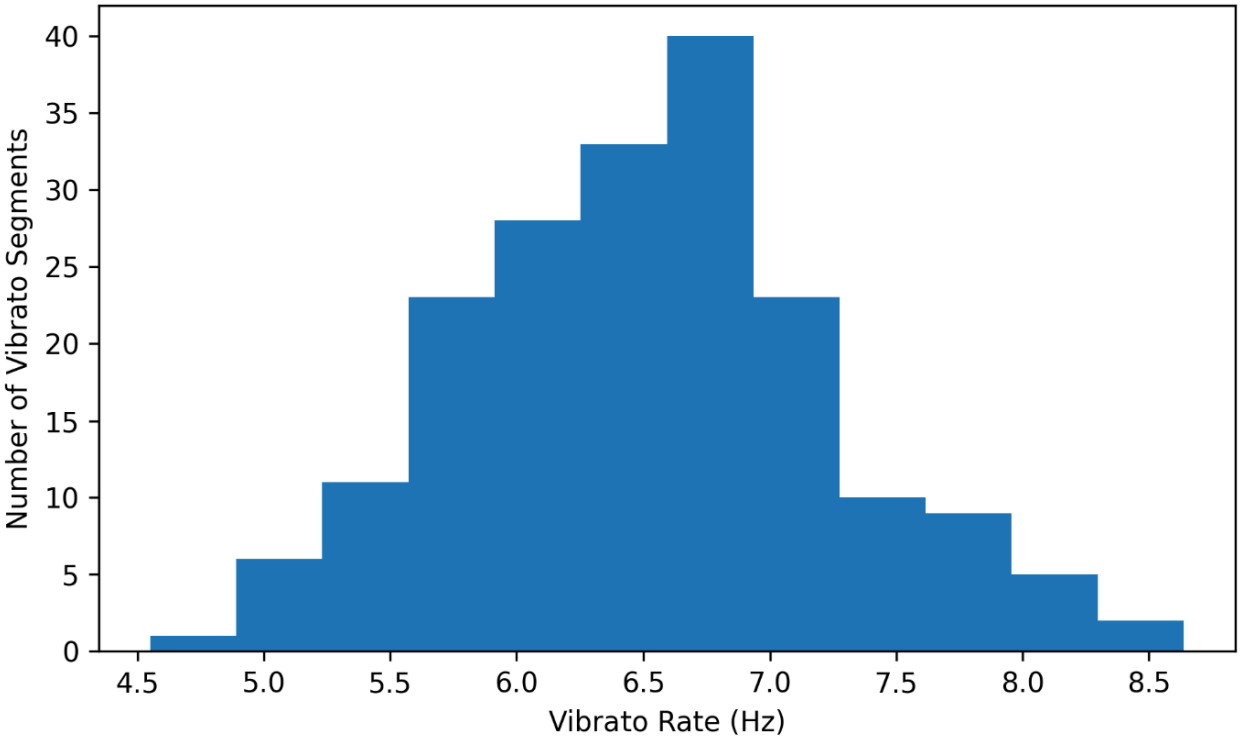

Figure 1. Distribution of estimated vibrato rates. Distribution of the estimated vibrato across the analyzed vibrato segments. Most vibrato segments showed rates between 5 and 8 Hz, and it was consistent with the descriptive statistics reported in the study.

Relatively consistent behavior was also demonstrated across recordings by pitch modulation. According to the fitted sinusoidal model, a mean vibrato amplitude was estimated to be 16.469 cents (SD = 6.155 cents) with a median value of 16.748 cents. The interquartile range was from 12.373 cents to 20.363 cents, while the observed range was from 2.046 to 38.517 cents. The corresponding mean peak-to-peak

Variable	Mean	SD	Median	95% Confidence Interval
Vibrato rate (Hz)	6.541	0.771	6.575	6.431–6.651
Vibrato amplitude (cents)	16.469	6.155	16.748	15.591–17.348
Peak-to-peak modulation (cents)	32.938	12.310	33.497	—

pitch modulation measured 32.938 cents (SD = 12.310 cents) with a median value of 33.497 cents (Table 2). Table 2. Descriptive statistics of violin vibrato. Descriptive statistics of the estimated vibrato characteristics across the analyzed violin recordings. For vibrato rate, vibrato amplitude, and peak-to-peak pitch modulation were presented with mean, standard deviation, median, and 95% confidence intervals.

A satisfactory representation of pitch modulation was provided by the sinusoidal mathematical model for most vibrato segments. Across the 191 analyzed segments, the mean coefficient of determination (R^2) was 0.718 (SD = 0.183), and the median R^2 was 0.755. The interquartile range for R^2 was from 0.638 to 0.838. Accordingly, the sinusoidal model explained an average of 71.8% of the detrended pitch variation, leaving an average of 28.2% unexplained. Based on the median R^2 of 0.755, the median proportion of unexplained variance was 24.5%. Unexplained variance ranged from 16.2% to 36.2% across the interquartile range of the model. In addition, the average root mean square error (RMSE) was 6.884 cents (SD = 2.731 cents). Therefore, the mean RMSE corresponded to around 41.8% of the mean vibrato amplitude of 1.469 cents. A total of 172 segments (90.1%) achieved R^2 to be greater than 0.50, whereas 121 segments (63.4%) achieved R^2 to be greater than 0.70 (Table 3). These results show that the sinusoidal model provided a moderate overall representation of the observed pitch modulation, while a meaningful proportion of variation remained unexplained.

Statistic	Value
Mean R^2	0.718
Median R^2	0.755
Mean RMSE (cents)	6.884
Segments with $R^2 \geq 0.50$	172 / 191 (90.1%)
Segments with $R^2 \geq 0.70$	121 / 191 (63.4%)
Segments with vibrato rate between 5–8 Hz	179 / 191 (93.7%)

Table 3. Performance of the sinusoidal mathematical model. Summary of the goodness-of-fit statistics for the sinusoidal mathematical model. Model performance was evaluated using the coefficient of determination, root mean square error, and the proportion of well-fitted vibrato segments. For each violin subset, descriptive statistics were also calculated. The Yangjian recordings recorded the highest average vibrato amplitude (18.689 cents), and an average vibrato rate of 6.659 Hz. The Laurel subset showed a mean vibrato rate of 6.595 Hz and average vibrato amplitude of 16.332 cents. The CMMSD recordings

Dataset	Segments	Mean Rate (Hz)	Mean Amplitude (cents)	Median R^2
Yangjian	81	6.659	18.689	0.753
Laurel	59	6.595	16.332	0.745
CMMSD	32	6.717	13.348	0.758
Coler2011	19	5.581	12.690	0.794

reported a mean vibrato rate of 6.717 Hz with lower average vibrato amplitude of 13.348 cents. Furthermore, the Coler2011 subset reported the lowest average vibrato rate (5.581 Hz) and an average vibrato amplitude of 12.690 cents (Table 4).

Table 4. Results by violin dataset. Comparison of vibrato characteristics among the four violin subses analyzed in this study. The table summarizes the number of analyzed segments, mean vibrato rate, mean vibrato amplitude, and median model goodness-of-fit for each dataset.

Fitted vibrato rates were compared with manually annotated half-cycle measurements available in the Yangjian subset to assess the mathematical model. A total of 84 vibrato segments were included in the validation analysis for which both manual annotations and fitted estimates were available. A strong positive relationship was observed between the two measurements, and a Person correlation coefficient was recorded as 0.810 ($p < 0.001$) (Figure 2). In addition, the mean absolute error between fitted and manually annotated vibrato rates was 0.300 Hz, and the root mean square error was 0.465 Hz. The manually annotated vibrato rate had an average of 0.232 Hz that was underestimated by the fitted model.

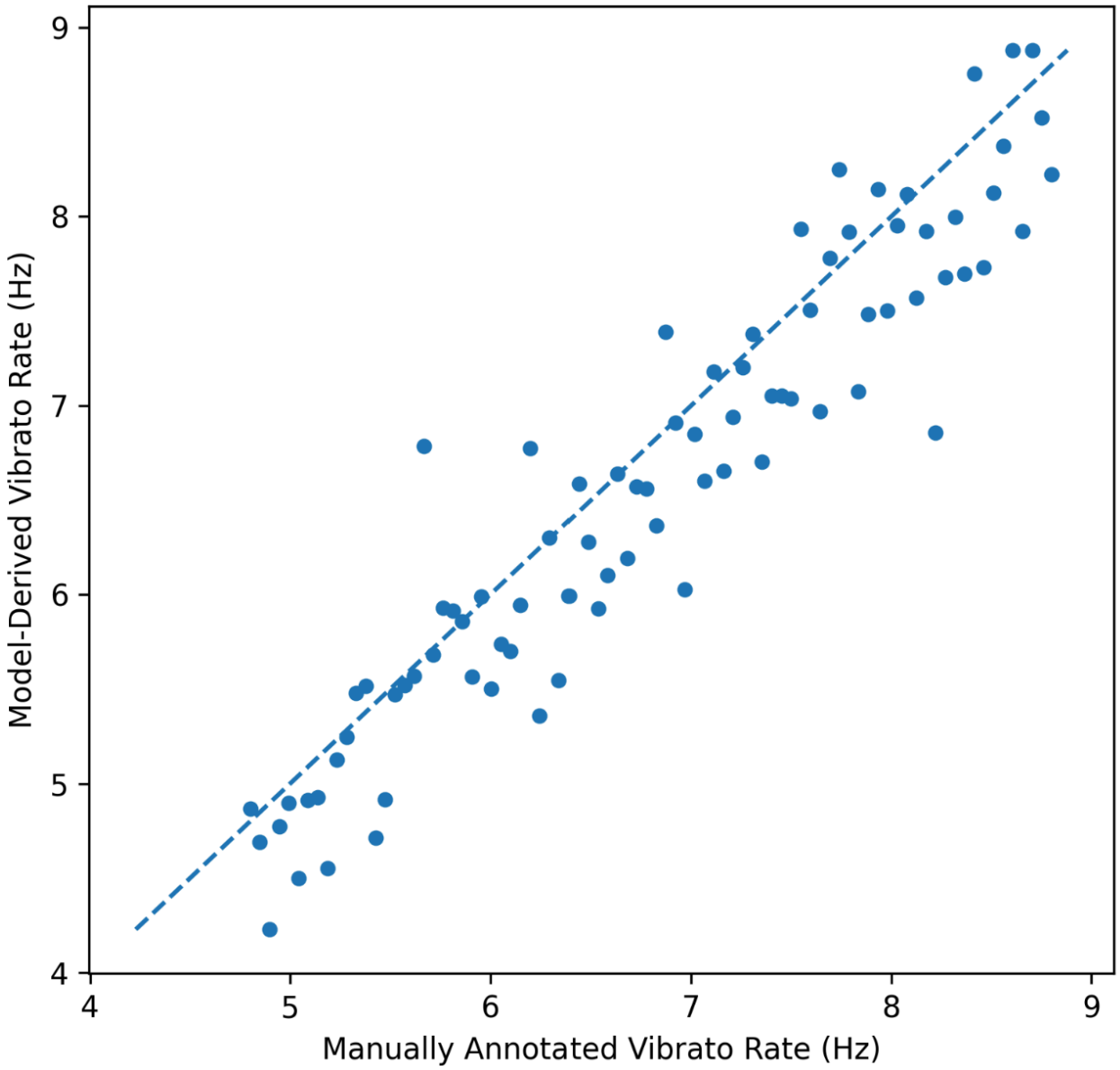

Figure 2. Validation of the sinusoidal mathematical model. Relationship between manually annotated vibrato rates and sinusoidal model-derived vibrato rates for the validation dataset. The strong positive association shows good agreement between the mathematical model and the manual vibrato annotations.

The association between vibrato rate and vibrato amplitude was examined by an exploratory analysis across all analyzed segments. The Pearson correlation coefficient between these variables was 0.236 ($p = 0.001$). This showed a weak positive relationship. Larger pitch modulation amplitudes was exhibited by segments with slightly faster vibrato in spite of substantial variability remaining across recordings.

Overall, violin vibrato in the analyzed recordings showed relatively consistent vibrato rates and amplitudes according to the quantitative analysis. Furthermore, a satisfactory representation of pitch modulation was provided by the sinusoidal mathematical model for most vibrato segments. This was reflected by the generally high R^2 values as well as the strong segment between fitted and manually annotated vibrato rates.

DISCUSSION

This study showed that violin vibrato may be effectively characterized with a relatively simple quantitative model through sinusoidal pitch modulation. Analyzing the 191 vibrato segments, the mean vibrato rate was 6.541 Hz, with nearly 94% of the segments showing vibrato rates between 5 and 8 Hz. This was consistent with the frequency range reported in previous acoustical studies (Brown & Vaughn, 1996; MacLeod, 2008). Furthermore, the sinusoidal model reported a mean R^2 value of 0.718, with 90.1% of the analyzed segments producing R^2 values of at least 0.50. This suggests that periodic mathematical functions demonstrate a reasonable approximation of vibrato behavior in the analyzed recordings.

In addition, the reliability of the mathematical model was further supported by the validation analysis. Comparison between model-derived rates and manually annotated half-cycle measurements indicated a strong positive correlation ($r = 0.810$, $p < 0.001$) with relatively small prediction errors (MAE = 0.300 Hz; RMSE = 0.465 Hz). As reported by these findings, the fitted sinusoidal model provided reasonably consistent estimates of vibrato rate, even though it did not capture all pitch variation within individual vibrato segments. The mean R^2 of 0.718 shows that around 28.2% of detrended pitch variation remained unexplained on average, while the RMSE of 6.884 cents showed around 41.8% of the mean vibrato amplitude. Therefore, the model should be not be interpreted as a complete description of its pitch dynamics but as a useful but simplified representation of violin vibrato. Therefore, these findings exhibited how mathematical modeling and computational analysis may be applied to examine an expressive musical technique with publicly available datasets.

In this study, the value of interdisciplinary research was demonstrated by integrating concepts from music performance, acoustics, physics, and mathematics. Instead of assessing vibrato solely through subjective musical interpretation, measurable acoustic variables and mathematical functions were used to quantify the vibrato in this study. With this approach, an accessible example of how mathematical modeling may be utilized to examine artistic phenomena was provided, while using computational methods commonly used in acoustics and signal processing.

However, several limitations should be acknowledged. First, only 17 publicly available violin recordings were analyzed in this study. Therefore, the findings represent the characteristics of the analyzed dataset rather than all violin performances. Second, many vibrato segments were originated from the same recordings. This means that observations may not be completely independent. Third, approximately periodic oscillation was assumed to intentionally simplify vibrato in the sinusoidal model, but real violin performances may show subtle irregularities, changes in bowing technique, expressive timing, and performer-specific characteristics that were not modeled. Finally, performer demographics, instrument

characteristics, or recording conditions were not included in the dataset. This may have prevented examination of factors that may influence vibrato behavior.

It is recommended for future research to substantially analyze larger violin datasets, compare performers, examine differences among musical styles, or examine additional string instruments, such as the viola and cello. Furthermore, more advanced quantitative models with nonlinear or time-varying oscillations may also improve descriptions of vibrato while still preserving the interdisciplinary connection among mathematics, physics, and music.

CONCLUSION

This study demonstrated that a simple sinusoidal quantitative model can capture a substantial, but incomplete, proportion of pitch modulation in violin vibrato. A total of 191 annotated vibrato segments from a publicly available violin dataset were analyzed, finding that vibrato rates were generally consistent. The mean value of it was 6.541 Hz, and the sinusoidal model explained 71.8% of detrended pitch variation on average, while 28.2% remained unexplained. The reliability of this quantitative approach was further supported by the strong agreement between model-derived vibrato rates and manually annotated measurements.

This study also illustrates the importance of interdisciplinary studies integrating music, mathematics, and acoustical physics. With openly available data and accessible computational techniques, this study showed that artistic performance may be examined through objective mathematical analysis. This study contributes to both the quantitative study of violin acoustics and an example of how quantitative modeling may be applied to real-world creative phenomena.

It is recommended for future study to examine more diverse dataset in larger scale, include more bowed string instruments, and explore more advanced mathematical models that may represent subtle variations in expressive performance. This work may further improve understanding of the physical and musical characteristics of vibrato and also strengthen the connection between STEM disciplines and the performing arts.

REFERENCES

Brown, J. C., & Vaughn, K. V. (1996). Pitch center of stringed instrument vibrato tones. The Journal of the Acoustical Society of America, 100(3), 1728–1735. doi:10.1121/1.416070.

de Cheveigné, A., & Kawahara, H. (2002). YIN, a fundamental frequency estimator for speech and music. The Journal of the Acoustical Society of America, 111(4), 1917–1930. doi:10.1121/1.1458024.

Fletcher, N. H., & Rossing, T. D. (1998). The physics of musical instruments (2nd ed.). Springer. doi:10.1007/978-0-387-21603-4.

Fritz, C., Woodhouse, J., Cheng, F. P.-H., Cross, I., Blackwell, A. F., & Moore, B. C. J. (2010). Perceptual studies of violin body damping and vibrato. The Journal of the Acoustical Society of America, 127(1), 513–524. doi:10.1121/1.3266684.

Geringer, J. M., MacLeod, R. B., & Allen, M. L. (2010). Perceived pitch of violin and cello vibrato tones among music majors. Journal of Research in Music Education, 57(4), 351–363. doi:10.1177/0022429409350510.

MacLeod, R. B. (2008). Influences of dynamic level and pitch register on the vibrato rates and widths of violin and viola players. Journal of Research in Music Education, 56(1), 43–54. doi:10.1177/0022429408323070.

Mellody, M., & Wakefield, G. H. (2000). The time-frequency characteristics of violin vibrato: Modal distribution analysis and synthesis. The Journal of the Acoustical Society of America, 107(1), 598–611. doi:10.1121/1.428326.

Rossing, T. D., Moore, F. R., & Wheeler, P. A. (2002). The science of sound (3rd ed.). Addison Wesley.

Seashore, C. E. (1938). Psychology of music. McGraw-Hill.

Song, W., Dinh, A. D., & Horner, A. B. (2024). The emotional characteristics of the violin with different pitches, dynamics, and vibrato. Proceedings of Meetings on Acoustics, 55(1), Article 035004. doi:10.1121/2.0001989.

Song, W., Law, M. H., Dinh, A. D., & Horner, A. B. (2025). Influence of pitch, dynamics, and vibrato on the emotional characteristics of violin sounds. The Journal of the Acoustical Society of America, 157(4 Suppl.), A311. doi:10.1121/10.0038176.

Song, W., Zhou, Z., Huang, Z., & Horner, A. (2024). A comparative study of violin and erhu emotional characteristics: Influence of playing techniques and instrument. Proceedings of Meetings on Acoustics, 54(1), Article 035002. doi:10.1121/2.0001921.

von Coler, H., & Lerch, A. (2014). CMMSD: A data set for note-level segmentation of monophonic music. In Proceedings of the AES 53rd International Conference on Semantic Audio. Audio Engineering Society.

von Coler, H., & Röbel, A. (2011). Vibrato detection using cross correlation between temporal energy and fundamental frequency. In Proceedings of the 131st Audio Engineering Society Convention. Audio Engineering Society.

Yang, L., Rajab, K. Z., & Chew, E. (2017). The filter diagonalisation method for music signal analysis: Frame-wise vibrato detection and estimation. Journal of Mathematics and Music, 11(1), 42–60. doi:10.1080/17459737.2016.1263897.