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Fall Voltage as a Key Parameter in MPD Thruster Performance Modeling

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ABSTRACT

Magnetoplasmadynamic (MPD) thrusters are a class of electric propulsion devices capable of producing high exhaust velocities and thrust through the control of magnetic fields and electric currents. These thrusters are of particular interest for deep-space missions due to their ability to operate effectively for extended durations. A key aspect of MPD thruster performance is the fall voltage, a voltage drop associated with plasma sheaths at the cathode and anode interfaces that represents energy lost as electrons cross electrode surfaces. In many system-level models, fall voltage is treated as a constant, but this simplification may introduce inaccuracies in predicting performance, particularly at lower discharge currents. This study investigates the sensitivity of MPD thruster performance to fall voltage using a simplified self-field MPD thruster model implemented numerically in Python. By varying fall voltage across a range of 20–60 V while holding other parameters constant, we evaluate impacts on thrust, discharge voltage, exhaust velocity, and efficiency over a discharge current range of 0–1000 A. Results demonstrate that while fall voltage does not influence thrust or exhaust velocity in the electromagnetic framework, it significantly reduces thruster efficiency. A 40 V increase in fall voltage (20 V to 60 V) produced approximately 65–66% reduction in predicted efficiency at both 200 A and 800 A. Efficiency trends predicted by the model are qualitatively consistent with experimental data from the Princeton University full-scale benchmark thruster reported by Nada [8], supporting the physical relevance of the sensitivity structure identified here. These findings underscore the importance of accurately modeling sheath behavior in MPD performance predictions and suggest that constant fall voltage assumptions may lead to substantial efficiency overestimation, particularly in low-current operational regimes.

1. INTRODUCTION

Magnetoplasmadynamic (MPD) thrusters are a class of electric propulsion devices capable of producing high exhaust velocities and thrust using the control of magnetic fields and electric currents [1]. Space missions such as deep space travel and human exploration of Mars are of particular interest for these thrusters. MPD thrusters, in contrast to chemical thrusters, can function effectively for extended periods of time, supplying the acceleration required for deep-space travel.

The fall voltage, a small voltage drop associated with the plasma sheaths at the cathode and anode, is a key aspect of MPD thruster performance. It represents the energy lost as electrons cross the electrode

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surfaces. In many system-level models, such as the Modified Maecker model [2], fall voltage is treated as a constant. However, this generalization may produce inaccuracies in predicting thrust, exhaust velocity, and efficiency, particularly at lower currents where plasma is only partially ionized.

The main purpose of this study is to investigate the sensitivity of MPD thruster performance to fall voltage. By varying the fall voltage in a simplified model of a self-field MPD thruster, the study evaluates how different assumptions impact thrust, voltage, exhaust velocity, and efficiency. Simulations using Python and numerical calculations allow for easy visualization of these effects and give insight into the importance of accurately modeling fall voltage in MPD analysis. The results of this study aim to inform future modeling and the design of MPD thrusters for mission applications.

2. THEORETICAL BACKGROUND

2.1 Thruster Operation

Magnetoplasmadynamic (MPD) thrusters are electric propulsion devices that use electromagnetic forces to accelerate plasma. In a typical MPD thruster, propellant gas is inserted between a central cathode and a surrounding anode. According to Ampere's law, when a large electric current flows through the ionized propellant, it generates a magnetic field [3]. The current density in the plasma and this magnetic field produce a force that accelerates the plasma out of the thruster, generating thrust.

The mechanism responsible for this acceleration is the Lorentz force:

$$F = q(E + v \times B) \qquad (1)$$

where q represents particle charge, E represents the electric field, v represents the particle velocity, and B represents the magnetic field. In an MPD thruster, the plasma consists of an extremely large number of charged particles, so behavior is modeled using a fluid representation. The electromagnetic force density is then expressed as [4]:

$$f = \rho_e E + J \times B \qquad (2)$$

where ρ_e is the net charge density and J is the current density. In MPD thrusters, the plasma is quasineutral, so $\rho_e \approx 0$, and the force simplifies to [5]:

$$f \approx J \times B \qquad (3)$$

In self-field MPD thrusters, the discharge current flowing between the cathode and anode generates a magnetic field. The product of the current density and this magnetic field is a force directed along the thruster axis, accelerating plasma downstream and generating thrust.

2.2 Performance Metrics

The most important performance quantities for this study are thrust, exhaust velocity, mass flow rate, voltage, and efficiency. Thrust (T) represents the force produced by the thruster. Mass flow rate ($\dot{m}$) describes the amount of mass passing through the thruster per unit of time [6]. Exhaust velocity (u_e) is related to thrust by:

$$u_e = T / \dot{m} \qquad (4)$$

Thruster efficiency (η) is defined as the fraction of input power converted into directed kinetic energy of the exhaust [5,7]:

$$\eta = Tu_e / VI \qquad (5)$$

where V is the total discharge voltage and I is the current. Efficiency is very sensitive to voltage losses, making accurate modeling of voltage essential for MPD thruster performance.

3. MODELING FRAMEWORK

3.1 Simplified Self-Field MPD Thruster Model

To investigate the sensitivity of MPD thruster performance to fall voltage, a simplified self-field MPD thruster model is used. The total discharge voltage across an MPD thruster can be expressed as:

$$V = V_{e\square} + V_{o\square\square}^{ec} + V^{i}_{a\square\square} \qquad (6)$$

where $V_{e\square}$ represents the electromagnetic acceleration voltage, $V_{o\square\square}^{ec}$ represents resistive (ohmic) losses, and $V^{i}_{a\square\square}$ represents the combined cathode and anode fall voltage. This study isolates the effect of $V^{i}_{a\square\square}$ by treating it as a variable parameter within a fixed thruster model.

3.2 Electromagnetic Thrust Model

In a self-field MPD thruster, thrust is primarily generated through J × B. Under simplifying assumptions, the total thrust scales with the square of the discharge current:

$$T = KI^2 \qquad (7)$$

where I is the discharge current and K is a proportionality constant that depends on thruster geometry and magnetic field distribution.

3.3 Exhaust Velocity and Mass Flow

Using the definition of exhaust velocity, the exhaust velocity can be expressed in terms of discharge current as:

$$u_e = KI^2 / \dot{m} \qquad (8)$$

For the purposes of this study, the mass flow rate ($\dot{m}$) is treated as constant, isolating the effect of fall voltage on performance metrics.

3.4 Efficiency Formulation

Substituting the expressions for thrust, exhaust velocity, and voltage into Eq. (5):

$$\eta = K^2I^3 / \dot{m}(aI + V^{i}_{a\square\square}) \qquad (9)$$

This equation shows that fall voltage directly reduces efficiency by increasing the denominator without contributing to thrust. The effect is greatest at low discharge currents.

3.5 Assumptions and Limitations

To focus the analysis on the influence of fall voltage, the following assumptions are made:

- The plasma is quasineutral, meaning the ionized gas remains electrically neutral.
- Mass flow rate is constant and independent of discharge current.
- Thruster geometry remains fixed.
- A discharge current range of 0–1000 A is applied to preserve realistic parameters.

These assumptions constrain the scope of the analysis in several important ways. If mass flow rate were allowed to vary with discharge current, which is expected when ionization fraction and propellant utilization change with power level, the denominator in Eq. (9) would become current-dependent, altering both the shape and magnitude of predicted efficiency curves. Similarly, if plasma properties such as electrical conductivity or degree of ionization were treated as functions of current rather than fixed constants, the effective resistance term in the ohmic voltage contribution would vary, further modifying predicted discharge voltages and efficiencies. These effects are expected to be most significant at low discharge currents, where partial ionization and variable mass utilization are most pronounced. The present analysis should therefore be interpreted as an idealized sensitivity study rather than a quantitative performance prediction; the qualitative trends identified are expected to remain physically meaningful, but the absolute efficiency values and the magnitude of fall voltage sensitivity may differ in a more complete model.

4. METHODOLOGY

This study evaluates MPD thruster performance sensitivity to fall voltage by varying V_{fall} across a range of 20–60 V while holding other parameters constant. Performance metrics include thrust, discharge voltage, exhaust velocity, and efficiency as functions of discharge current over a range of 0–1000 A.

The governing equations were implemented numerically in Python 3 using the NumPy library for computation and Matplotlib for visualization. Multiple fall voltage values (20 V, 40 V, and 60 V) were computed and plotted across the same current range to enable direct comparison.

To contextualize the model's efficiency predictions against experimental data, results were compared qualitatively with efficiency measurements reported by Nada [8] for the Princeton University full-scale benchmark thruster operating with argon propellant at approximately 4.7 g/s. Because that thruster operates in the 1–30 kA regime, a direct quantitative comparison is not appropriate; however, the trend of efficiency rising with current before approaching an onset-limited peak is consistent with the behavior predicted by the simplified model at lower currents.

Model parameters were set as: $K = 2\times10^{-6}$ N/A², $\dot{m} = 0.02$ kg/s, and $a = 0.002\ \Omega$.

5. RESULTS

The simplified self-field MPD thruster model was evaluated over a discharge current range of 0–1000 A for three assumed fall voltages: 20 V, 40 V, and 60 V. All results were generated numerically in Python and are presented in Figs. 1 and 2.

5.1 Thrust Behavior

Thrust increases quadratically with discharge current, consistent with the relationship $T = KI^2$. All three fall voltage cases produce identical thrust curves, confirming that fall voltage does not directly influence electromagnetic thrust generation and acts as an electrical loss mechanism rather than a thrust-producing contribution.

5.2 Discharge Voltage

The total discharge voltage increases linearly with current for each case, with vertical offsets corresponding to the selected fall voltages. Increasing fall voltage shifts the voltage–current curve upward without changing its slope, confirming that fall voltage increases total electrical input power at every current level.

5.3 Exhaust Velocity

Since exhaust velocity is directly proportional to thrust under constant mass flow rate ($u_e = T/\dot{m}$), it also increases quadratically with discharge current. Exhaust velocity remains unaffected by fall voltage assumptions, consistent with the electromagnetic scaling framework of the model.

5.4 Efficiency Trends

Thruster efficiency exhibits strong sensitivity to fall voltage, particularly at low discharge currents. At lower current levels, the constant fall voltage term dominates the denominator of Eq. (9). As a result:

- Higher fall voltage significantly reduces efficiency in the low-current regime.
- The efficiency curves for different fall voltages diverge most strongly at low current.
- At higher currents, the term aI becomes large compared to V^{i}_{a}□□, reducing the relative impact of fall voltage.

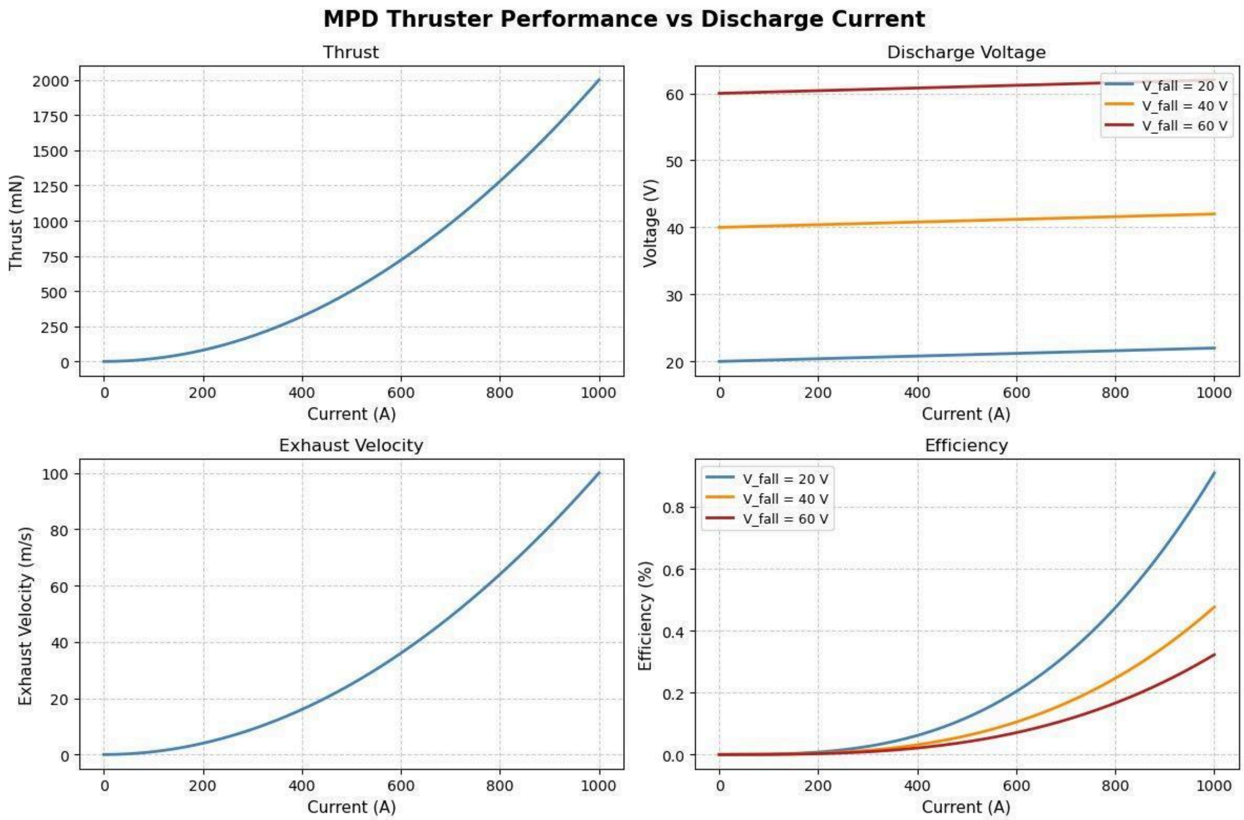

Fig. 1 MPD thruster performance metrics as a function of discharge current for fall voltage values of 20 V, 40 V, and 60 V. (a) Thrust, (b) Discharge Voltage, (c) Exhaust Velocity, (d) Efficiency. Model parameters: $K = 2\times10^{-6}$ N/A², $\dot{m} = 0.02$ kg/s, $a = 0.002\ \Omega$

5.5 Comparison with Experimental Literature

To contextualize these findings, the efficiency trends predicted by the simplified model were compared qualitatively with experimental data reported by Nada [8] for the Princeton University full-scale benchmark thruster operating with argon propellant at approximately 4.7 g/s (Fig. 2). That thruster operates in the 1–30 kA regime, making direct quantitative comparison inappropriate. However, both the model and the experimental data exhibit the same fundamental trend: efficiency rises nonlinearly with discharge current and is sensitive to voltage losses in the system. Nada [8] reports a peak efficiency of approximately 19% at 18 kA, beyond which onset phenomenon limits further performance gains. This behavior is qualitatively consistent with the model's prediction that efficiency gains diminish as the electromagnetic voltage contribution grows relative to the fall voltage term. Specifically, the qualitative trend that can be meaningfully compared is the nonlinear rise in efficiency with increasing discharge current, which both the simplified model and the Nada [8] data display across their respective operating ranges. The gross shape of this efficiency curve is physically rooted in the same mechanism: as discharge current increases, the electromagnetic voltage term grows relative to the fixed voltage losses (fall voltage in the model; sheath and resistive losses in the experiment), driving efficiency upward. In contrast, several aspects of the experimental results cannot be meaningfully compared with the present model. The absolute efficiency values differ substantially: the model predicts efficiencies well below 1% in the 0–1000 A range, while Nada [8] reports efficiencies up to approximately 19% in the kiloampere regime, reflecting the much higher electromagnetic power levels at which the benchmark thruster was designed to operate. Additionally, the onset phenomenon observed in the experimental data, which limits performance gains at high currents, has no analog in the simplified model, which does not account for plasma instabilities or electrode erosion effects that govern onset behavior.

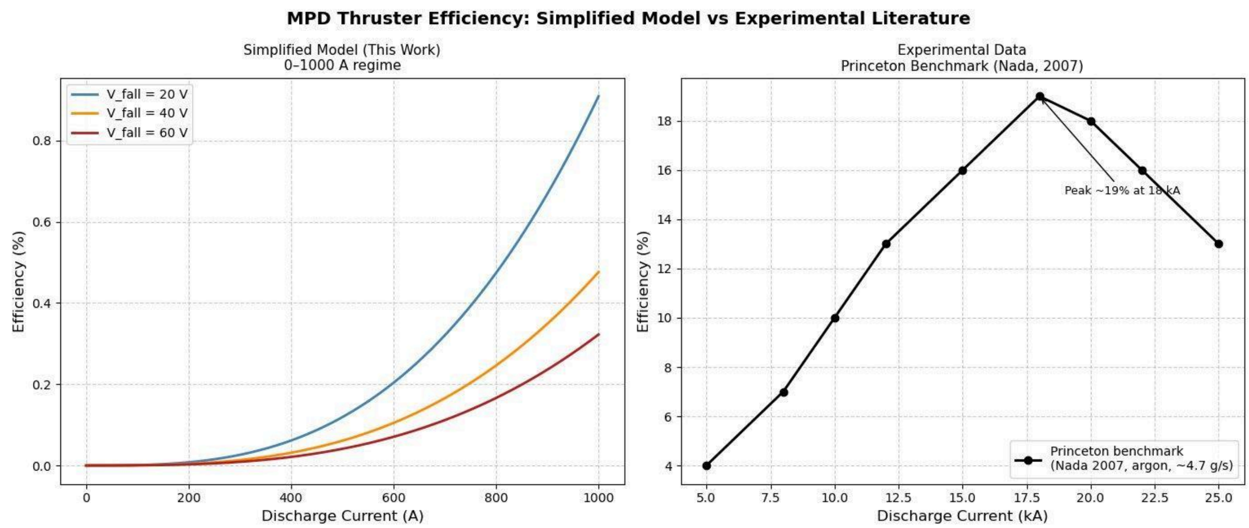

Fig. 2 Comparison of efficiency trends between the simplified model (left, this work) and experimental data from the Princeton University full-scale benchmark thruster using argon propellant at approximately 4.7 g/s (right, digitized from Nada [8]). Note differing current scales between panels

6. QUANTITATIVE ANALYSIS

To quantify the impact of fall voltage on efficiency, values of I = 200 A and I = 800 A are evaluated using Eq. (9) with the selected model parameters. The numerator term $K^2 = (2\times10^{-6})^2 = 4\times10^{-12}$. The denominator simplifies to:

$$\dot{m}(aI + V_{a}^{i}\square\square) = 0.00004I + 0.02V_{a}^{i}\square\square \qquad (10)$$

6.1 Case 1: I = 200 A

Numerator: $4\times10^{-12} \times (200)^3 = 3.2\times10^{-5}$

Denominators: 20 V → 0.408; 40 V → 0.808; 60 V → 1.208

Efficiencies: 20 V → $\eta = 7.84\times10^{-5}$; 40 V → $\eta = 3.96\times10^{-5}$; 60 V → $\eta = 2.65\times10^{-5}$

Percent reduction (20 V → 60 V): $(7.84\times10^{-5} - 2.65\times10^{-5}) / 7.84\times10^{-5} \approx 66\%$

6.2 Case 2: I = 800 A

Numerator: $4\times10^{-12} \times (800)^3 = 2.048\times10^{-3}$

Denominators: 20 V → 0.432; 40 V → 0.832; 60 V → 1.232

Efficiencies: 20 V → $\eta = 4.74\times10^{-3}$; 40 V → $\eta = 2.46\times10^{-3}$; 60 V → $\eta = 1.66\times10^{-3}$

Percent reduction (20 V → 60 V): $(4.74\times10^{-3} - 1.66\times10^{-3}) / 4.74\times10^{-3} \approx 65\%$

7. CONCLUSIONS

This study investigated the influence of fall voltage on the predicted performance of a simplified self-field magnetoplasmadynamic (MPD) thruster model. By isolating fall voltage as a variable parameter and holding thruster scaling, mass flow rate, and voltage coefficients constant, the analysis evaluated its impact on thrust, exhaust velocity, discharge voltage, and efficiency. All results were generated using a reproducible Python-based numerical simulation.

The results demonstrate that fall voltage does not affect thrust or exhaust velocity within the simplified electromagnetic framework, as thrust is governed solely by the J × B interaction and scales quadratically with discharge current. However, fall voltage directly increases total discharge voltage and therefore electrical input power.

Quantitative analysis revealed that increasing fall voltage from 20 V to 60 V produced approximately a 65–66% reduction in predicted efficiency at both 200 A and 800 A. This reduction occurs because fall voltage contributes to input power without contributing to electromagnetic thrust production, and remains a significant portion of total discharge voltage across the examined current range.

Qualitative comparison with experimental efficiency data from Nada [8] for the Princeton University full-scale benchmark thruster confirms that the fundamental trend — efficiency rising nonlinearly with current and remaining sensitive to voltage losses — is physically consistent across current regimes, lending credibility to the sensitivity structure identified in the simplified model despite the difference in operating scale.

These findings indicate that assuming constant or underestimated fall voltage values in MPD performance models may lead to large overprediction of thruster efficiency, though it is important to note that these conclusions are drawn from a simplified analytical framework and have not been validated against experimental measurements. Future work should focus on incorporating current-dependent fall voltage behavior and validated sheath models to improve performance predictions for high-power electric propulsion systems.

DECLARATIONS

Funding: No funding was received for conducting this study.

Conflicts of interest: The author has no competing interests to declare that are relevant to the content of this article.

Data availability: The data generated during this study are included in the figures presented in this article.

Code availability: The Python simulation code supporting this study is available from the author upon reasonable request.

Author contributions: Samay Goyani conceived the study, developed the methodology, performed all analysis, and wrote the manuscript.

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