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Visualizing the Effect of Dark Energy on Cosmic Expansion

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ABSTRACT

This project investigates how radiation density, matter density, and dark energy affect the expansion of the universe using the Friedmann equation. We built a Python simulation that visualizes how the scale factor and expansion curves evolve over cosmic time when varying matter, radiation, and dark energy densities. The final simulations showed that the universe was first dominated by radiation, then by matter, and now by dark energy. We verified the consistency of these results across different methods. Building on the results, we extended the project to include dynamical dark energy $w(a) = w_0 + w_a(1 - a)$. Instead of assuming dark energy was perfectly constant, we implemented two new parameters: w0 and wa. To connect the model with observations, we constructed a Hubble diagram using Type Ia supernova data from Pantheon+ and SH0ES. The model successfully demonstrates the sequence of radiation-, matter-, and dark energy-dominated eras, illustrating how early expansion slows under matter and accelerates later due to dark energy.

INTRODUCTION

The expansion of the universe has long held the attention of cosmological research. When Edwin Hubble proved, in the 1920s, that galaxies moved away from one another, this discovery opened up an entirely new set of questions about the universe's beginnings (Smith, 1990). What forces shape the universe's expansion? Will the universe expand forever? How fast is the universe expanding? Subsequent findings—including the cosmic microwave background (CMB) and, in 1998, observations of distant Type Ia supernovae—show that the universe's expansion is accelerating (Riess et al., 1998). Together, these findings indicated that the universe's expansion is governed by two opposing forces: the gravitational attraction of matter and the counteracting push of dark energy. Dark energy, which makes up roughly 70% of the universe today, remains one of the greatest mysteries in physics (Workman et al., 2023). Unlike ordinary matter or radiation, dark energy appears to accelerate expansion instead of slowing it. This expansion is described mathematically by the Friedmann equation (Peebles & Ratra, 2003). The first equation relates the expansion rate to the energy contents of the universe, while the second determines whether the expansion accelerates. The Friedmann equation provides the framework for both observations and the simulations in this project.

Recent observational projects continue to refine these questions. In 2025, the Dark Energy Spectroscopic Instrument (Gialamas et al., 2025), a project managed by the Lawrence Berkeley National Laboratory, released its first results from mapping millions of galaxies. DESI measures galaxy redshifts to build a 3D model of the universe, which helps estimate the effects of dark energy. These results hinted that dark energy might not be a constant property of space, but one that changed slightly over time. While the evidence does not overturn the standard Λ Cold Dark Matter (ΛCDM) model, it does suggest that evolving dark energy models may better match the data (Gialamas et al., 2025). Around the same time, the European Space Agency (ESA) launched the Euclid space mission (Euclid Collaboration, 2025). Euclid examines galaxy clustering and weak gravitational lensing from space, providing another means to test whether Dark Energy evolves. Together, these projects suggest there should be further investigation into whether dark energy is truly constant or has changed over the history of the universe. To connect the model to data, we used Type Ia supernovae from the Pantheon+ and SH0ES compilation to build a Hubble diagram, which relates redshift to luminosity distance and allows for direct comparison between theoretical predictions and observational data (Brout et al., 2022).

We wanted to understand how changing different cosmic parameters affects our universe's expansion history and age. Specifically, we looked at how tweaking dark energy's properties (w) and the amounts of matter (Ω_m) and dark energy (Ω_Λ) would change the universe's expansion history and age. We figured that having more matter would slow expansion, while more dark energy would speed it up. We also hypothesized that radiation would only matter in the very beginning of the universe. To test these ideas, we turned the Friedmann equation into an interactive simulation that shows exactly what happens when you adjust these cosmic factors.

We also wanted to test if different numerical strategies—ODE integration versus Akima interpolation—would give consistent results. This served as a stability check to verify that the model's behavior is not an artifact of one particular method. We expected that they would agree, but the Akima interpolation might be faster and smoother than the ODE solver.

Originally, my model assumed a constant dark energy equation of state ($w = 1$), which is the standard ΛCDM model. After DESI's 2025 results, suggesting Dark Energy might change over time, we extended the project by adding two parameters

$$w(a) = w_0 + w_a(1 - a)$$

(Workman et al., 2023). My new hypothesis expected $w_a > 0$ to weaken past dark energy, causing later acceleration, and $w_a < 0$ to strengthen it, leading to earlier acceleration. During the universe's earlier history, all models should look the same, but they should diverge at later times when dark energy dominates.

LITERATURE REVIEW & BACKGROUND

Cosmic history is divided into three stages. First came the radiation-dominated era, lasting approximately 50 000 years after the Big Bang, during which photons carried most of the energy (Peebles & Ratra, 2003). This era was followed by the matter-dominated era, extending from about 50 000 years after the Big Bang until around 9 billion years later, when galaxies began to form (Peebles & Ratra, 2003). In today's cosmic landscape (the last 5 billion years), we find ourselves in an era where dark energy takes the lead, driving the universe's accelerating expansion (Peebles & Ratra, 2003).

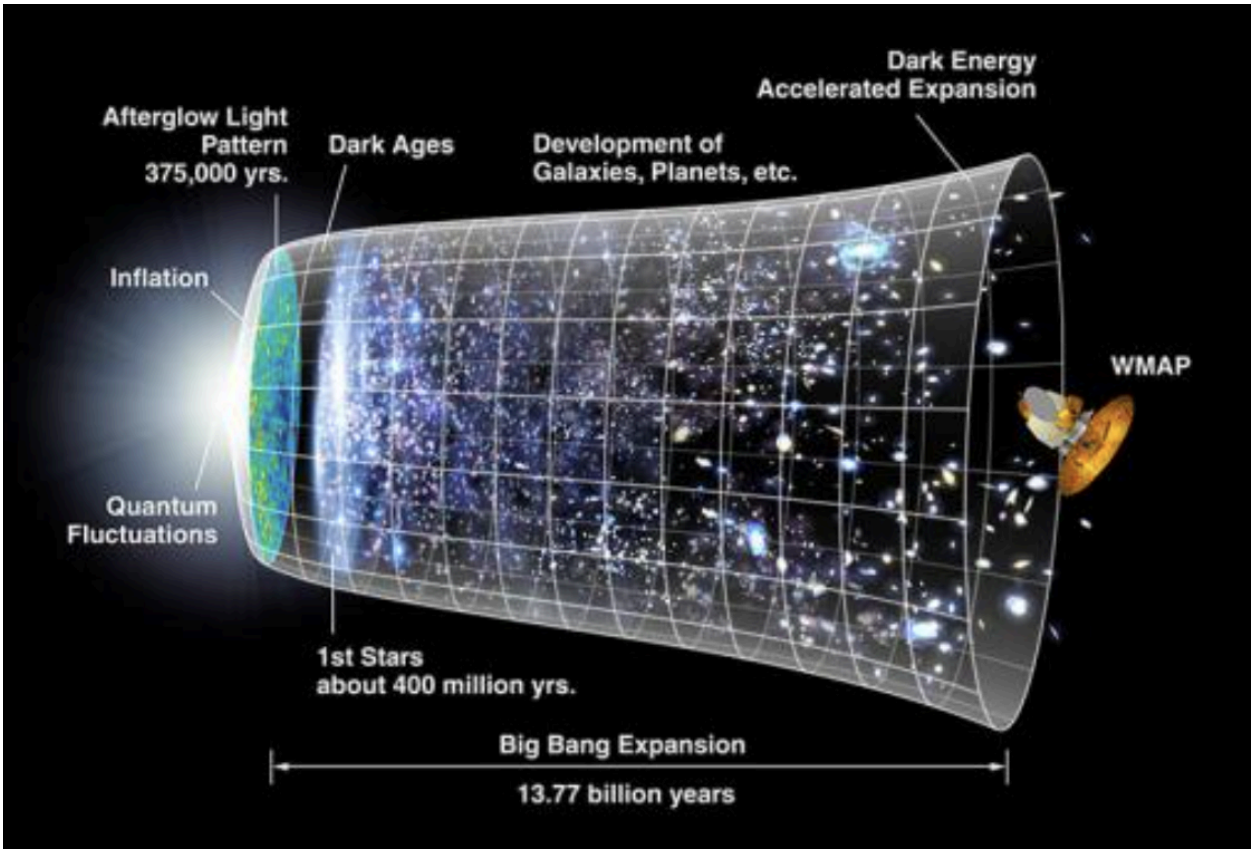


Figure 1. Schematic of the universe's expansion history, from inflation to the current dark energy dominated era. Adapted from the Center for Astrophysics, Harvard & Smithsonian (Dark energy and dark matter).

Friedmann equations connect Einstein's general relativity with the large-scale behavior of the universe. They describe how the scale factor, $a(t)$, evolves depending on the density of matter, radiation, and dark energy (Peebles & Ratra, 2003). Working with the cosmological principle, which assumes a homogeneous and isotropic universe, they form the basis of the standard cosmological model (Peebles & Ratra, 2003).

The Friedmann equation is:

$$H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda a^{-3(1+w)}} \quad (2)$$

This equation reveals which term dominates at different points in time. The term $H(a)$ is the Hubble parameter, which tells us how fast the universe is expanding at a given time. The constant H_0 is the Hubble constant today, the expansion rate right now. Each Ω term represents a density parameter, which expresses the fraction of the universe's total energy density contributed by a specific component: matter (Ω_m), radiation (Ω_r), curvature (Ω_k), and dark energy (Ω_Λ). Radiation has the steepest dependency on a, scaling as a^{-4}, so it dominates when the scale factor is $a < 10^{-4}$ (roughly until 50 000 years after the Big Bang). Matter takes over after radiation fades, and dark energy dominates when it grows larger.

Here, $a(t)$ represents the scale factor, the relative size of the universe, where $a = 1$ today and $a < 1$ in the past. The Hubble parameter $H = \dot{a}/a$ describes the rate of cosmic expansion. Each energy component—matter (Ω_m), radiation (Ω_r), and dark energy (Ω_Λ)—contributes differently to the total energy density. The equation of state parameter w expresses the pressure-to-density ratio for dark energy. In the extended model, $w(a) = w_0 + w_a(1 - a)$ allows w to evolve over time, while redshift $z = v/c$ relates recessional velocity to the speed of light.

METHODOLOGY

We built the model in Google Colaboratory[1] and relied on standard scientific tools for computation, specifically ODE integration and plotting. We used various libraries, including NumPy, Matplotlib, iPyWidgets, and SciPy (Harris et al., 2020; Matplotlib Developers; Virtanen et al., 2020; Akima, 1970). Simulations vary the density parameters Ω_m, Ω_r, Ω_Λ , and the dark-energy equation-of-state parameter (w).

The extended model refers to the version of my simulation where dark energy is allowed to vary with the scale factor (w_0 and w_a). For the extended model, we modified the solver so that w became a function of the scale factor rather than a constant and added interactive sliders for w_0 and w_a. This lets us explore different models: standard ΛCDM when $w_0 = -1$, and $w_a = 0$ dark energy when $w_a \neq 0$.

The governing equation solved was:

$$H(a) = H_0\sqrt{\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda a^{-3(1+w)}}$$

[1] colab.google.cooverlappedm

It should be noted that numerically, we cannot start the system exactly at $a = 0$ because terms a^{-3} a^{-4} will diverge to infinity, so we will begin our integrals from small positive values. Apart from the numerical aspect, the Big Bang itself is defined as the limit $a \to 0$, $t \to 0$, so we cannot fully capture the singularity, since physical theories break down at $t = 0$ this point. This is done to make the equations numerically stable and the plots easier to interpret.

To implement the model correctly, the units had to align. For example, the Hubble constant usually appears in units such as kilometers per second per megaparsec. These work for observation, but not inside a numerical model, because all quantities must be expressed in consistent units. In practice, this meant the constant required conversion into "per second." For plotting purposes, we expressed the time output in $Gigayears$, which made the graph easier to interpret. This correction aligned the model with accepted values and also highlighted the importance of unit consistency.

At this point, we used two different numerical strategies to compute $a(t)$. The first relied on solve_ivp, which integrates

$$a = aH(a) \tag{3}$$

forward in time with timed steps (Virtanen et al., 2020). The second method turns the equation around and computes

$$\frac{dt}{da} = \frac{1}{aH(a)} \tag{4}$$

then uses an Akima spline to integrate and get $t(a)$ (Virtanen et al., 2020). Using both methods gave us a built-in cross-check because when the two curves from the methods overlapped, it confirmed the model behaved correctly.

In addition to the theoretical modeling, we also used observational data from the Pantheon+ and SH0ES supernova coalition to make the Hubble diagram (Figure 8). The key coding steps were:

```
z = df["zCMB"].values
mu = df["MU_SH0ES"].values
D_L = 10**((mu - 25.0)/5.0)
```

First, we take the redshift values from the data and store them in z. Next, we take the distance modulus values from the SH0ES dataset and store them in μ. Finally, we calculate the luminosity distance DL using the formula:

$$DL = 10^{\frac{(\mu - 25)}{5}} \tag{5}$$

We created a plot of velocity and luminosity distance (DL). The term DL is the luminosity distance, which tells us how far away an object is based on how bright it appears. The symbol μ is the distance, which compares an object's observed brightness with how bright it would actually be. These modeling

steps, combined with observational data from supernovae, provided the foundation for the results presented in the next section.

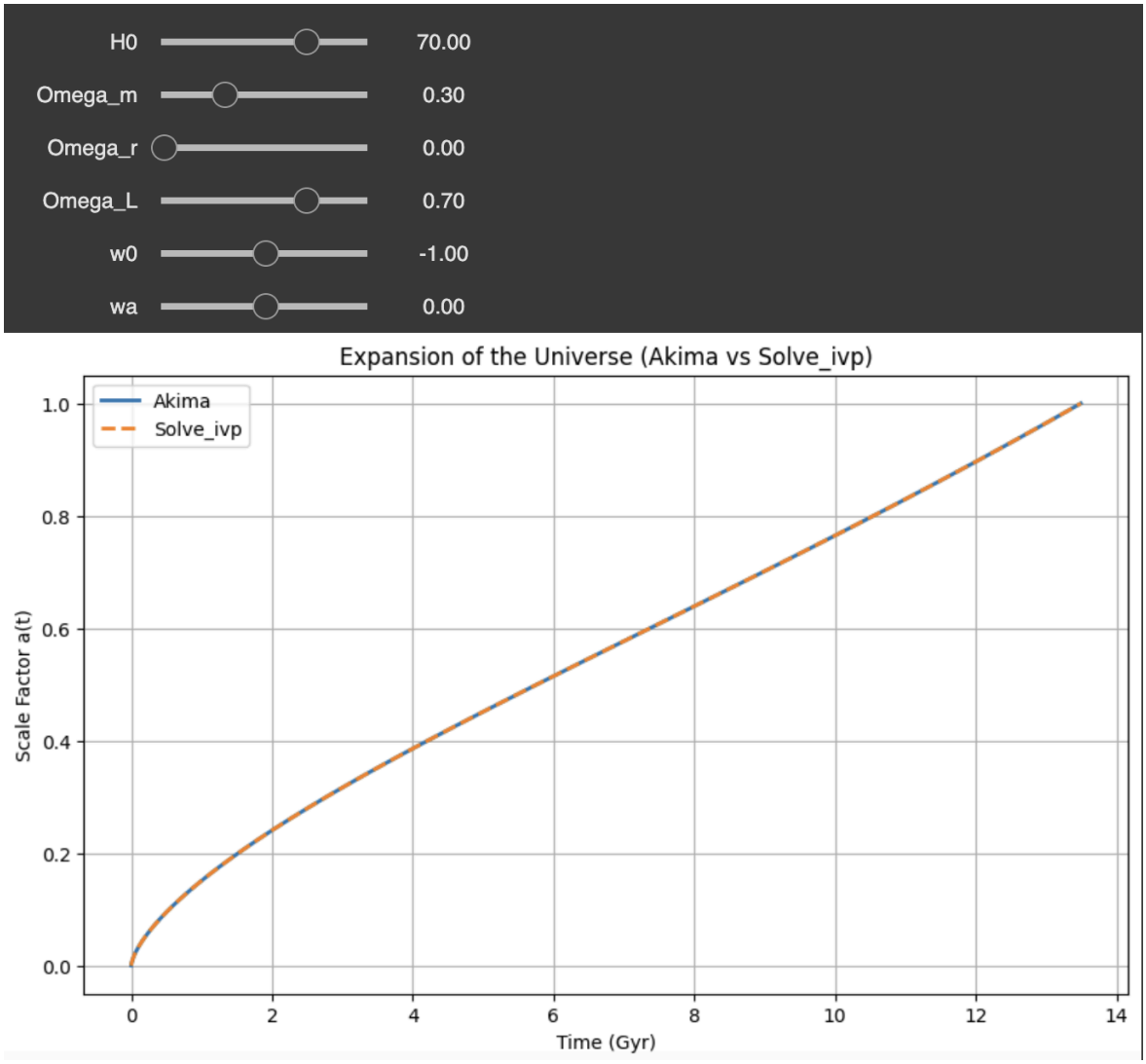
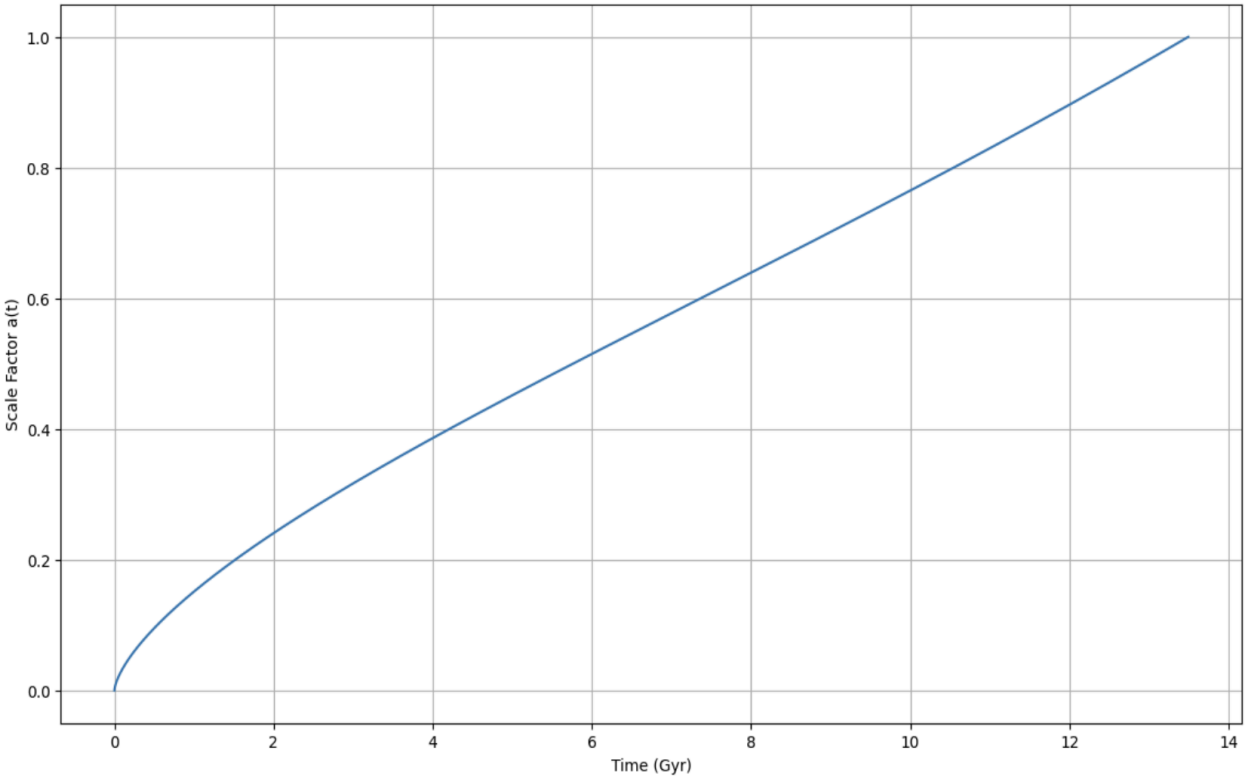


Figure 2. Google Colab environment with interactive sliders for H_0, Ω_m, Ω_Λ, Ω_r, w_0, and w_a.

RESULTS AND DISCUSSION

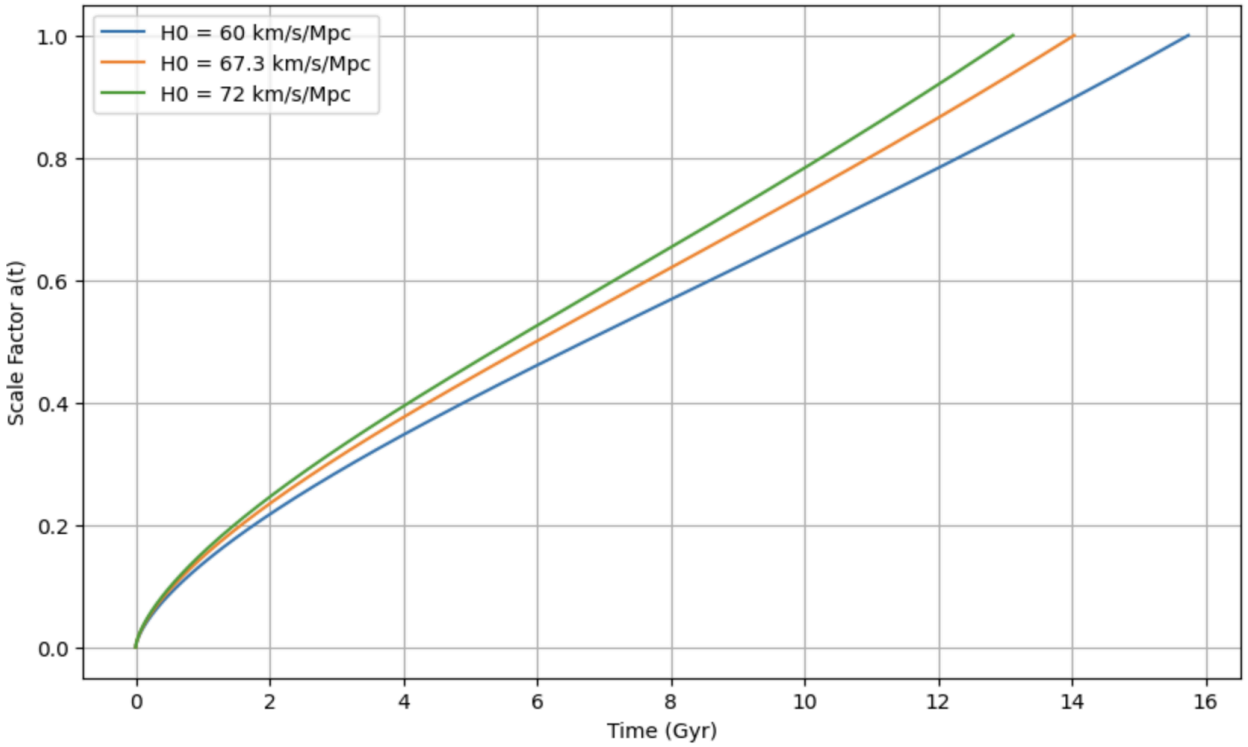
The simulations reproduced the expected sequence of cosmic eras, illustrating how radiation, matter, and dark energy each shaped expansion at different times. When radiation density dominated, the scale factor grew rapidly at very early times but quickly lost influence as matter became the primary driver. Once dark energy began to dominate, the model reproduced the expected late-time acceleration.

To explore these dynamics, we generated a series of simulations summarized in Figures 3–8. Each figure isolates a key parameter or comparison to illustrate how radiation, matter, and dark energy influence the universe's expansion.



Parameter	**Value**
Hubble constant (H_0)	70.00 $km/s/Mpc$
Matter density (Ω_m)	0.30
Radiation density (Ω_r)	0
Dark Energy density (Ω_Λ)	0.70
Dark energy equation of state (w)	-1

Figure 3. Baseline Expansion History. The curve begins near zero scale and reaches $a = 1$ today. The steady early rise reflects radiation, the mid-time slowdown reflects matter domination, and the late upward bend shows dark energy-driven acceleration.



Parameter	**Value**
Hubble constant (H_0)	(60.0, 67.3, 72.0) $km/s/Mpc$
Matter Density (Ω_m)	0.30
Radiation Density (Ω_r)	0
Dark Energy Density ($\Omega\Lambda$)	0.70
Dark energy Equation of State (w)	-1

Figure 4. Sensitivity to The Hubble Constant. A larger Hubble constant shifts the "today point" (where $a = 1$) to an earlier time. A smaller Hubble constant shifts it to a later time. Changing the Hubble constant rescales the time axis without changing the domination eras of the different species. After testing the sensitivity to the Hubble constant, we explored the role of matter density.

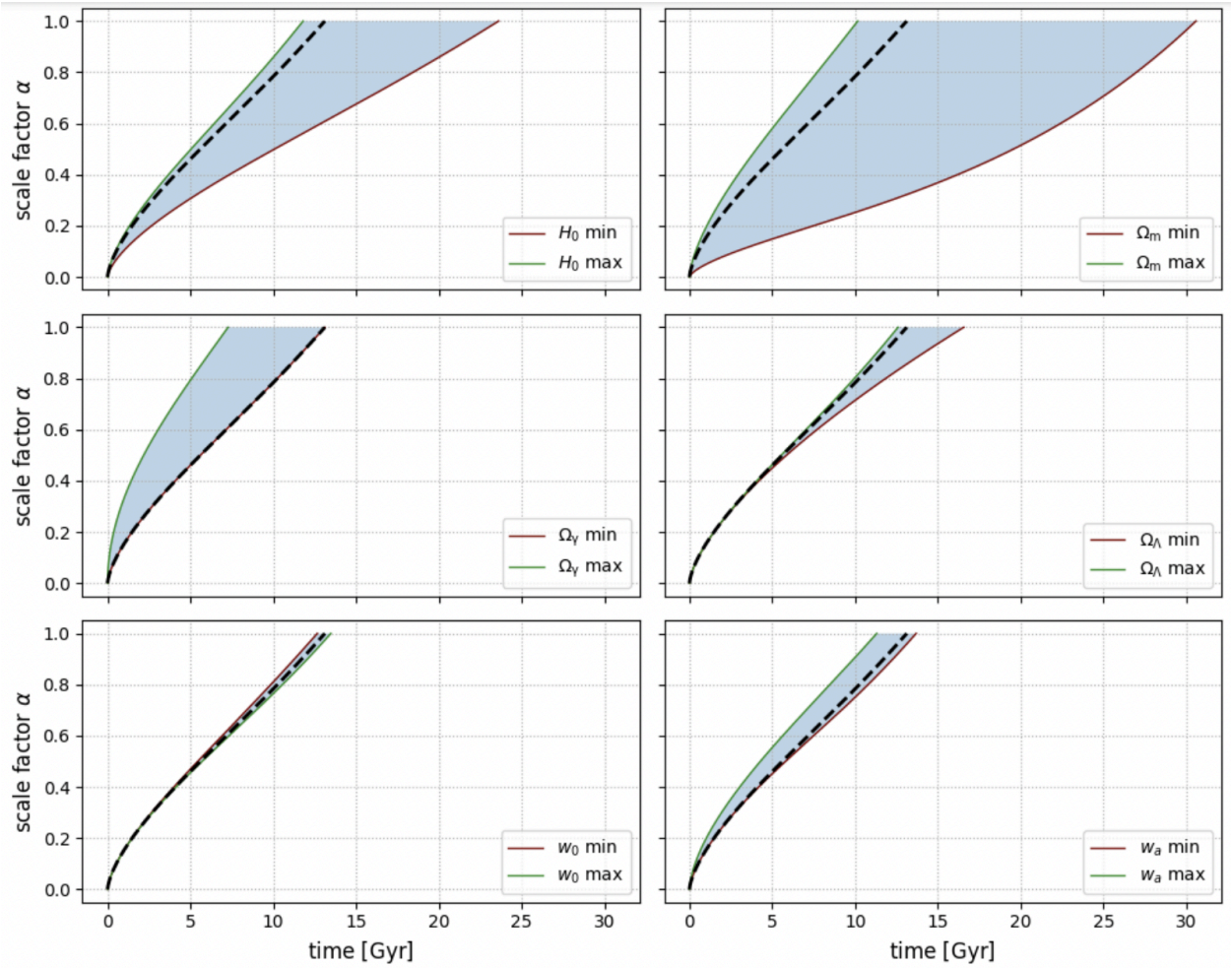


Figure 5. Altering cosmological parameters. Six subplots show the scale factor $a(t)$ vs. cosmic time (Gyr) while varying one parameter at a time around a baseline flat ΛCDM model (black dashed). For each panel, the red curve is the "min" value, the green curve is the "max" value, and the blue shading encloses the min–max envelope (possible age range).

In the top-left panel, larger H_0 compresses the time axis, reaching $a = 1$ sooner, while smaller H_0 stretches it. In the top-right panel, higher Ω_m delays the onset of acceleration, while lower Ω_m lets dark energy dominate earlier. In the middle-left panel, radiation mainly affects the earliest times, with increasing Ω_r steepening the very early rise before matter takes over. In the middle-right panel, higher Ω_Λ triggers earlier late-time acceleration, while lower Ω_Λ postpones it. In the bottom-left panel, changing today's dark-energy equation-of-state value, w_0, shifts the late-time acceleration while leaving early history nearly unchanged. In the bottom-right panel, varying w_a in $w(a) = w_0 + w_a(1 - a)$ alters the late-time slope.

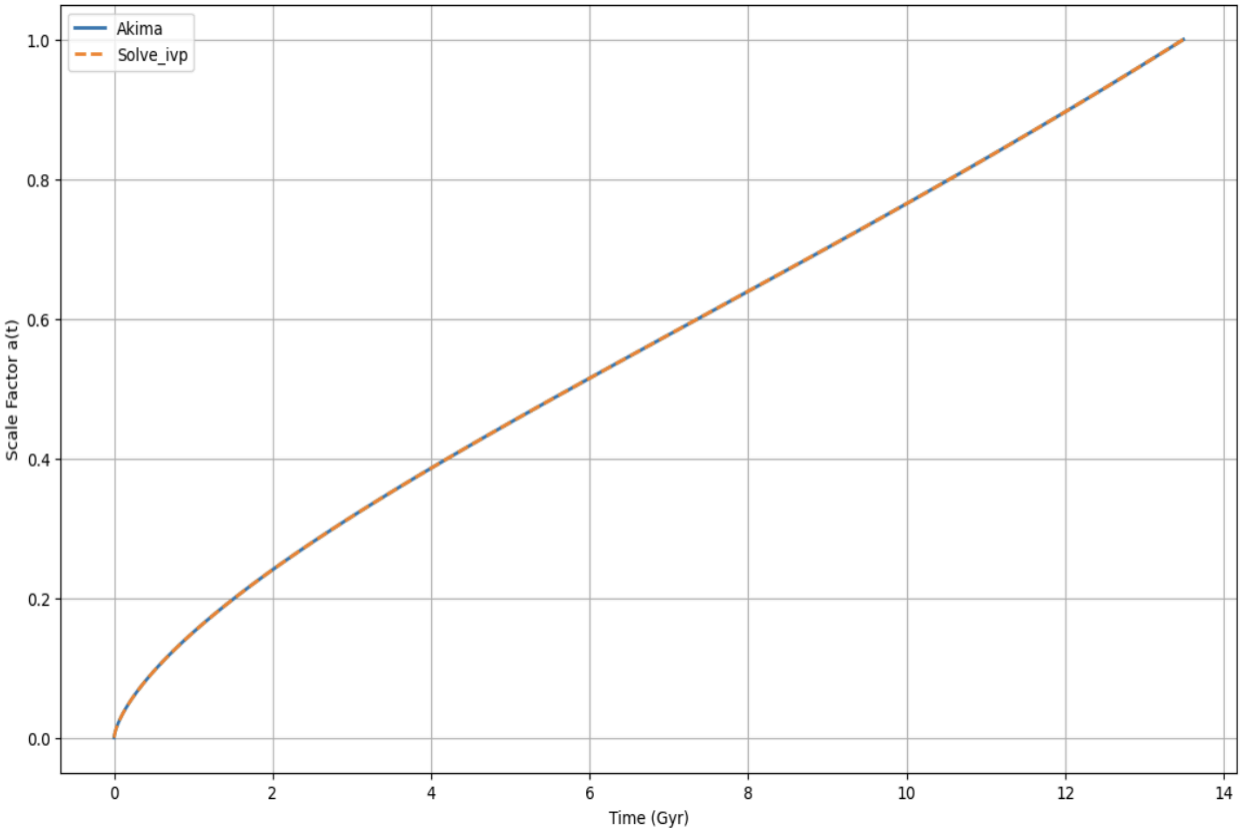


Figure 6. Akima vs. Solve_ivp. Both the Akima and Solve_ivp methods overlap, proving there were no errors in the code. With solver consistency established, we extended the model to explore evolving dark energy.

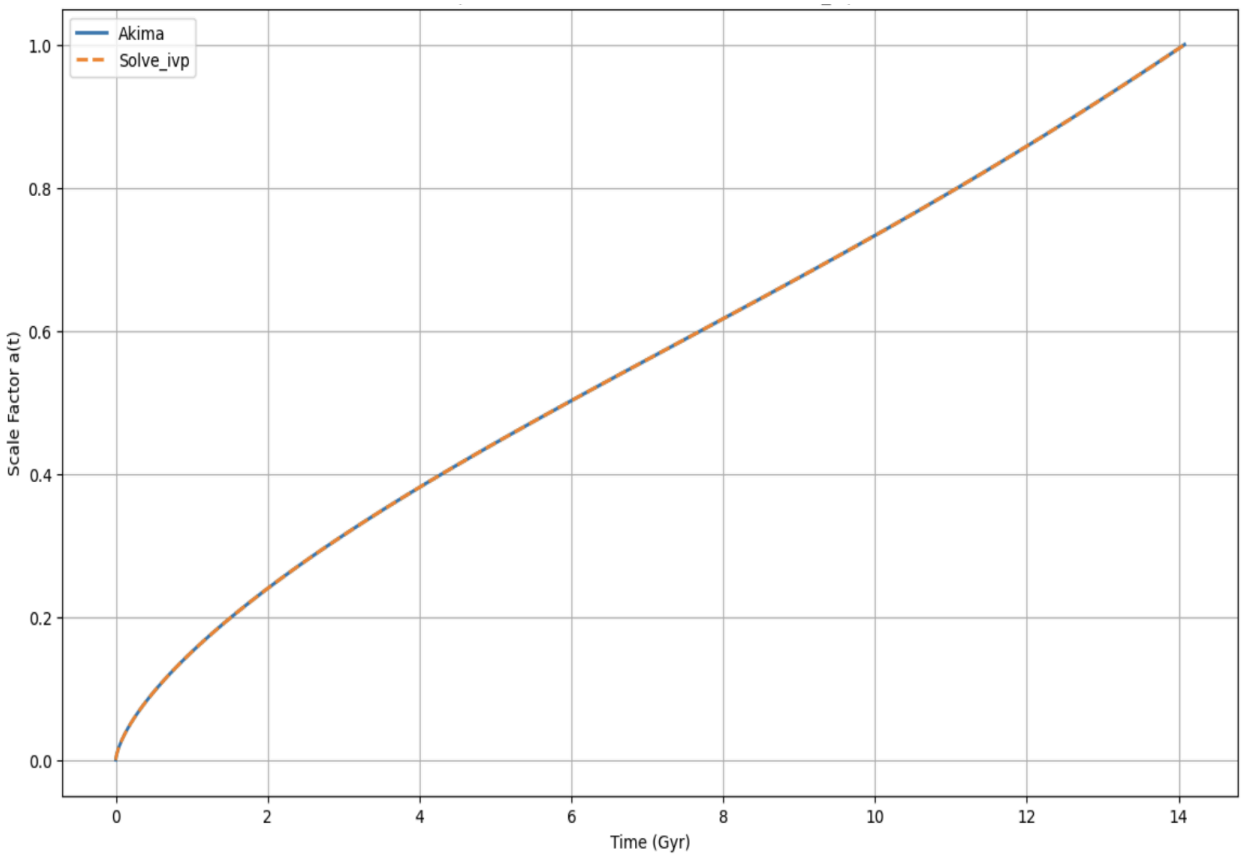


Figure 7a. Dynamical Dark Energy ($w_a = -1$). This figure depicts the effects of varying the w_a parameter. W_0 is set to -1 while w_a has values of -1.0. The model produced a longer age of about 14 Gyr

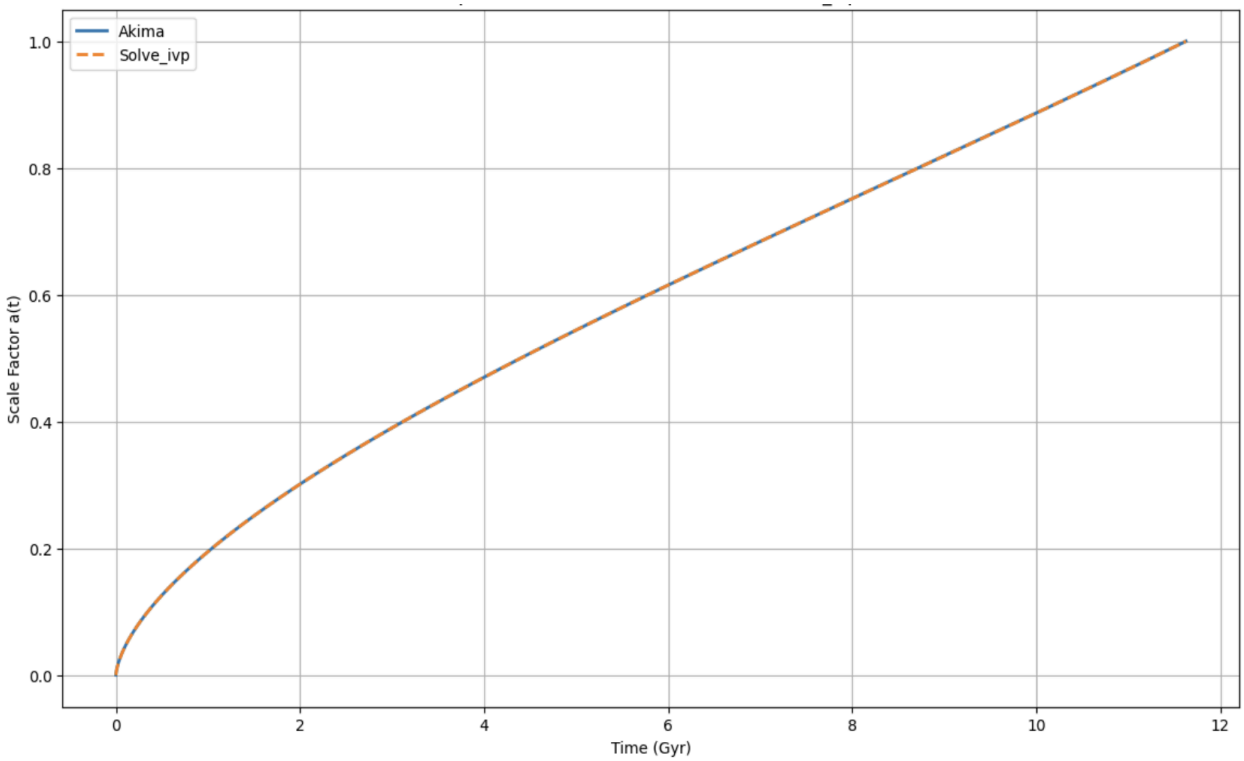


Figure 7b. Dynamical Dark Energy ($w_a = 1$). This graph also shows the effects of varying the w_a parameter. W_0 is set to -1 while w_a has values of 1.0. It reached today's scale factor in only 12 Gyr.

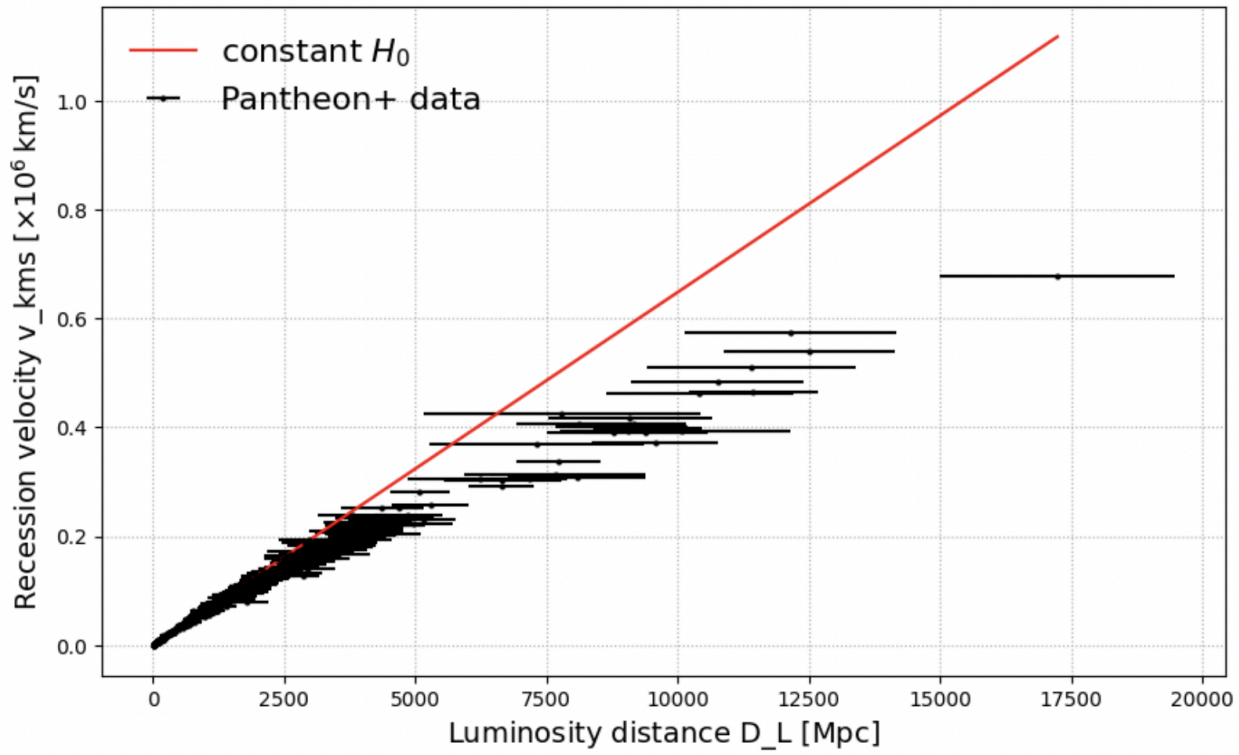


Figure 8. Hubble diagram from Pantheon+ and SH0ES Supernova Data. The black points show Type Ia supernovae from the Pantheon+ and SH0ES compilation, using recession velocity ($v = cz$) and luminosity distance (D_L). Each point has error bars representing observational uncertainties. The red line represents the best-fit line through the Pantheon+ data corresponding to a Hubble constant of $H_0 = 64.8 \pm 0.002\ km/s/Mpc$. Deviations from linearity at higher distances reflect the influence of dark energy on expansion.

The results in Figures 3-5 align well with the theoretical predictions of the standard cosmological model (Peebles & Ratra, 2003). In particular, radiation dominated the early universe. Matter shaped most of the universe's intermediate history (roughly from 50 000 years to around 9 billion years), before dark energy began to drive late-time acceleration. The onset of dark energy-driven acceleration, shown in Figures 3 and 7, occurs in the relatively recent universe (about the late 5 billion years).

The process highlighted how sensitive cosmological modeling remains to assumptions and computational details. Even small oversights, such as inconsistent units, can produce larger problems, such as unrealistic ages and expansion histories. Comparisons with established studies confirmed that the model successfully represents the expected behavior. The timing of the transitions between eras and the shape of the expansion curves matched the observational evidence, such as the cosmic microwave background and the Supernova measurements of accelerated expansion. The results demonstrate how relatively simple computational tools can still yield meaningful insights into solving the Friedmann equations.

The model does have limitations, however. It assumed a flat universe and neglected minor contributions from neutrinos and curvature. These effects are negligible in this context, and excluding them kept the model simple and efficient.

When comparing solvers, each showed distinct strengths. The ODE solver (solve_ivp) offered adaptive accuracy and automatically reduced step sizes in the early universe, while the Akima route produced perfectly smooth curves and required fewer lines of code. Their close agreement confirmed that the expansion histories reflected the underlying physics rather than the numerical artifacts.

The dynamical dark energy results behaved differently from my initial expectations. We predicted that a positive w_a would weaken dark energy in the past and delay acceleration, lengthening the universe's age, while a negative w_a would strengthen dark energy and shorten the universe's age. Instead, the opposite occurred: $w_a = 1.0$ produced a younger universe (~12 Gyr), while $w_a = -1.0$ produced an older one (~14 Gyr). After discussion with my mentor, we learned that this reversal comes from normalizing $a = 1$ today. Changing w_a effectively alters the "cosmic ruler," shifting ages in ways that are not intuitive, even though the late-time acceleration trends remain consistent with expectations.

To test the theoretical prediction regarding the Friedmann equation, astronomers use Type Ia supernova explosions. These explosions act as standard candles because of their known intrinsic brightness, making it possible to infer their distances via their observed luminosity and distinctive light curve decay rates (Riess et al., 1998).

These findings connect directly to current observational efforts for cosmology. Surveys such as DESI and Euclid are beginning to test whether $w = -1$ is sufficient, or whether evolving dark energy models of the form $w(a) = w_0 + w_a(1 - a)$ provide a better fit. Figure 8 shows this broader context: deviations from a linear Hubble relation at higher redshifts reflect the imprint of dark energy on cosmic expansion.

CONCLUSION

The Friedmann equations provided a conceptual framework for investigating how radiation, matter, and dark energy alter the universe's scale factor through time. The simulations reproduced the expected cosmic timeline: radiation domination in the early universe, matter domination during structure formation, and late-time acceleration driven by dark energy.

The project showed not only that relatively simple computational models can capture complex cosmological phenomena, but also that attention to detail is essential for reliable results. Solving the Friedmann equation using Akima and Solve_ivp gave two perspectives for the same equation, aiding in checking for correctness, while illustrating the strengths of each approach.

Finally, extending the model to dynamical dark energy demonstrated how even small changes to the equation can alter the inferred age of the universe. As new surveys continue to refine our measurements, computational experiments like this one highlight both the promise and the challenge of examining one of cosmology's deepest mysteries: the true nature of dark energy.

DATA AND CODE AVAILABILITY

CodingFriedmannEquation.ipynb

SH0ES_Data.ipynb

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