

Calculating the Escape Velocity and Lower Mass Bound of a Body within a Dark Matter Halo

Hriday Jha

ABSTRACT

Dark matter remains to be one of the most speculative and mysterious substances from our current perspective in astrophysics. This paper will aim to provide educated hypotheses on the fermionic particle which makes up dark matter, by calculating the lower bound of the particle mass in an average dwarf spheroidal galaxy. This bound can be obtained through deriving the Escape Velocity based on calculations regarding Yukawa forces between dark matter particles. This can be done using the Pauli Exclusion Principle regarding fermionic particles to determine the mass density of the galaxy. This mass density can be calculated using models of the Jeans Analysis and Stellar Kinematics employing the Poisson Equation and Newton's Shell Theorems.

INTRODUCTION

In our Milky Way, the supermassive black hole at the centre, Sagittarius A*, contributes only 0.01 percent of the mass of the whole galaxy. Yet every celestial body revolving around the galactic centre does so with immense speeds. Why? To answer this, physicists speculated the existence of a new substance, known as dark matter.

In order to provide detailed hypotheses as to what subatomic particle dark matter is made up of, certain properties of said particle have to be calculated. These include the mass, the charge, and the spin. The former is the most important, as it influences dark matter's gravitational force.

The identity of Dark Matter could solve many of astrophysics' most pondered questions. It could explain in greater detail how the universe originated. It could broaden our understanding of how the early universe was like, and how subatomic matter was first created.

METHODOLOGY

We first begin our calculation with the Jeans Equation, which allows us to determine the behaviour of subatomic particles under a gravitational force. We use the Jeans and Newton's Shell Theorems to obtain the Gravitational and Yukawa Potentials, the latter of which we have varied the strength and range.

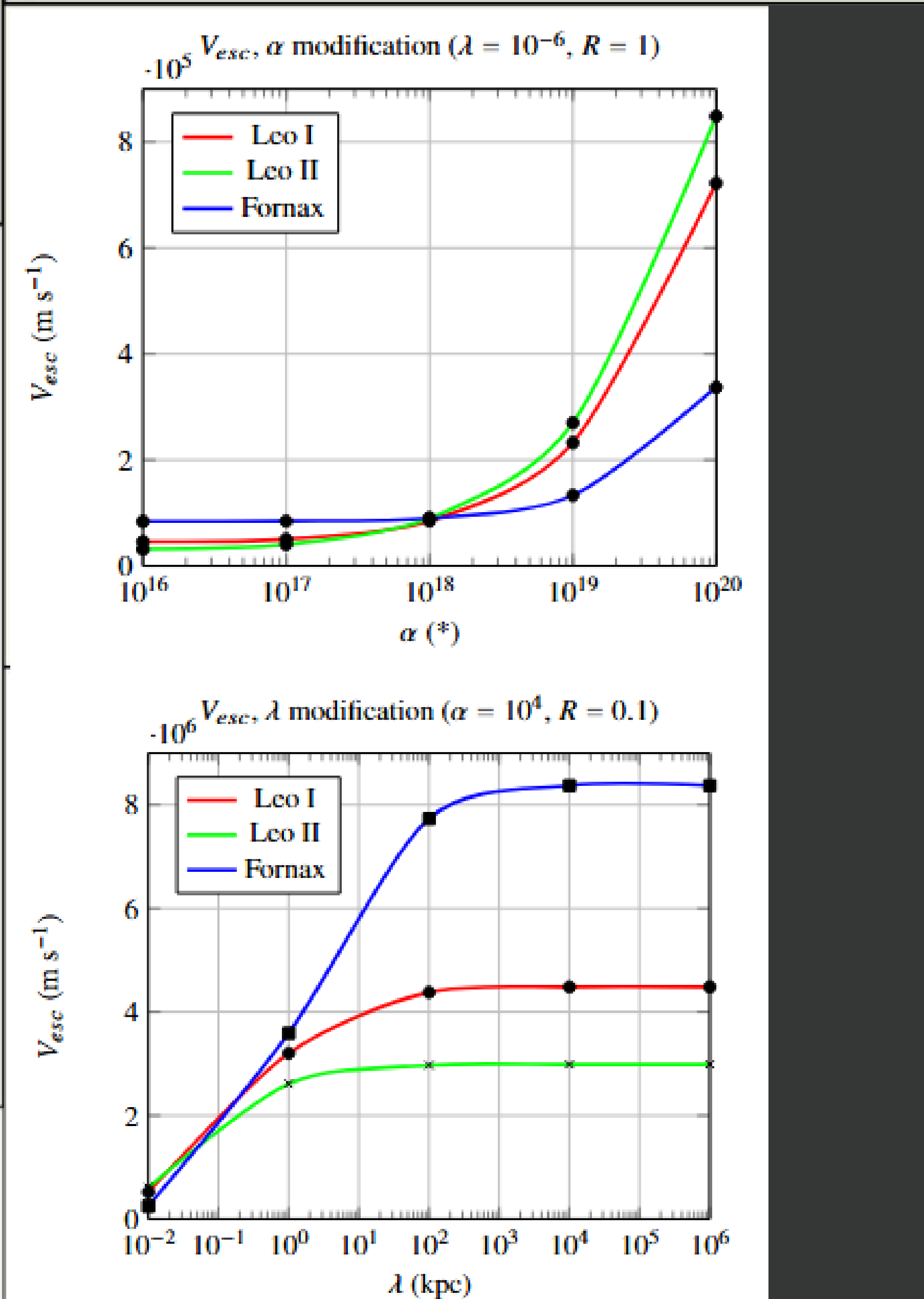
From here, we can simply use the Work-Energy Theorem to calculate the final escape velocity of a body within this halo. After obtaining the escape velocity, we can simply use Pauli's Exclusion Principle to obtain the Lower-Mass Bound (also known as the Gunn-Tremaine Bound) with respect to the halo's density profile.

After obtaining this final equation, we can simply code a program to obtain these escape velocity values. Here, we have used the repository given by Alvey et. al. which provides the standard radial and density profile values. Plugging in these values gives us the final result for the escape velocity of a given body within the halos of these galaxies.

RESULTS

When varying the strength of the Yukawa Force, the graph obtained is quasi-parabolic, with the escape velocity being approximately proportional to the square of the Yukawa strength.

When varying the range of the force, the graph obtained is asymptotic, hence the escape velocity is approximately proportional to the Yukawa range until a given point.



DISCUSSION

The results that have been obtained satisfy the general consensus of the Yukawa Force, with respect to Alvey et. al.'s repository. It is important to note that the absolute values vary greatly depending on the chosen radial and density profiles (alongside the constant value of the strength and range).

For instance, when varying the strength of the force, the range must be kept at a given value, i.e., from a range below 10^{-5} kiloparsecs. This is because below this value of range the escape velocity decreases rapidly, unable to be registered by python. Similar observations have been observed throughout the programming stage. The link for Alvey's repository can be found within the reference page.

CONCLUSION

We have calculated the escape velocity of a body inside a dark matter halo through two forces present between the particles: the Gravitational force, and the Yukawa force, the latter dependent on the net strength of the force and the range of the force. After coding the final integral equation, we can obtain values of the escape velocity. These values can be modified by changing the strength and range, and can thus derive a pattern as to how the mass of dark matter halos are reliant on these two parameters. Based on our findings, we have concluded that the strength and range follow quasi-parabolic and asymptotic relationships with the escape velocity respectively, where range is directly proportional to the escape velocity until it hits a certain limit.

Program Link:
<https://colab.research.google.com/drive/1M9Y2V1b3IQTeBs3dYQxj5jBXa2dat2Mp?usp=sharing>

BIBLIOGRAPHY

[1] Alvey et al., "New Constraints on the Mass of Fermionic Dark Matter from Dwarf Spheroidal Galaxies", 2020, p. 1-4
[2] Bogorad et al., "Coherent Self-Interactions of Dark Matter in the Bullet Cluster", 2025, p. 3, 8
[3] Di Paolo et al., "Phase Space Mass Bound for Fermionic Dark Matter from Dwarf Spheroidal Galaxies", 2018, p. 4-5
[4] Jo Bovy, "Dynamics and Astrophysics of Galaxies", chap. 2.1, 2.2, 5.3, 5.4