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Towards a Comprehensive Scale for Measuring Partisan Gerrymandering: Integrating Demographic Uniformity and Border Effects

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ABSTRACT

The study of the quantification of partisan Gerrymandering is still a contentious topic in mathematics and statistics. Though a host of techniques have been developed, each having certain strengths and weaknesses, no definitive universally applicable method has been arrived at. This study presents a succinct analysis of leading scales developed to quantify Gerrymandering, primarily focusing on mathematical foundations, empirical results, and weaknesses of each. This study borrows from a comparative study carried out in Warrington (2019) (Ref. 1) and Buzas-Warrington (2021) (Ref. 2) to arrive at leading strengths and weaknesses of scales like Efficiency Gap, Declination, Partisan Bias, and Mean Median Difference. This study uses appropriately developed scales to devise a new method, primarily focusing on demographic unity in neighboring geospatial districts, as a secondary consideration, focusing on border lengths. This method is devised to integrate leading strengths of existing scales developed, along with addressing primarily leading weaknesses, in recording finer levels of Gerrymandering that rely on demographic disunities. The method developed is presented in detail, outlining mathematical expressions, conceptual foundations, and empirical studies to validate. This study will present, through theoretical analysis and simulation studies, a method to quantify Gerrymandering as a far more precise index.

INTRODUCTION

Partisan gerrymandering-the manipulation of the boundaries of electoral districts for undue advantage toward a particular political party-threatens severely the integrity and representative nature of democracy. This problem is most pronounced in single-member district systems with plurality voting, since these contexts provide the greatest opportunities for strategic manipulation of district lines. Quantification is consequently the linchpin not only for the academic investigation of gerrymandering but also for practical policy-making-reviews by courts, redistricting commissions, and public advocacy.

The last two decades have seen a plethora of mathematical and statistical measures developed to detect and quantify partisan gerrymandering, including but not limited to the Efficiency Gap, Declination, Partisan Bias, and Mean-Median Difference. While all of these measures give various insights into the gerrymandering phenomenon, no consensus exists on which measure is necessary or sufficient for making determinations about unfair district plans. These measures also exhibit varying sensitivity to the distribution of partisan votes, the number and configuration of districts, voter turnout, and underlying geography.

Recent works have pointed out that there are limitations and shortcomings of existing methods, especially their proclivity toward favoring some types of asymmetry over others. To provide an example, a method that considers solely the vote share allocated to geographic districts might fail to detect instances of demographic and partisan outliers that cross district lines, as occurs with more cunning instances of gerrymanders. Likewise, a compactness measure that is centered on geographic district boundary irregularities might yield false positives or negatives, as irregularities are not necessary conditions of a partisan gerrymander.

In this work, the objective is to fill these voids by first presenting a comprehensive review of the current state of gerrymandering indices, assessing the mathematized underpinnings, performance, and restrictions of the current measures. Furthermore, based on the critique presented, a new measure is introduced that chiefly aims to emphasize the unity of demographic or partisan distribution in neighboring districts—a criterion that currently remains unexplored in available measures. The new measure aims to encompass both obvious and not-so-obvious gerrymandering, while simultaneously ensuring that the drawbacks of the previous measures be avoided.

The rest of the paper is organized as follows. Section 2 overviews the conceptual underpinnings of partisan gerrymandering and outlines the standards against which the fairness of district plans is assessed. Section 3 takes a comparative look at the existing quantitative measures, using substantial assertions from Warrington (2019) and Buzas & Warrington (2021). Section 4 describes the proposed measure, explaining its mathematical formulation and interpretative logic. Section 5 addresses the empirical strategy for the validation of the new measure, including the simulation-based approaches. Section 6 evaluates the potential implications and applications of the novel scale. Concluding remarks with possible further research avenues are presented in Section 7.

CONCEPTUAL FOUNDATIONS OF PARTISAN GERRYMANDERING

What Constitutes a Gerrymander?

In essence, partisan gerrymandering is the practice of manipulating electoral district boundary patterns to privilege one party over the other. It is usually accomplished by the use of two strategies: "packing" (pitting the votes of the rival party into as few districts as possible to minimize their representation elsewhere) and "cracking" (scattering the votes of the rival party across several districts to ensure that the rival party never reaches a majority in those districts).

Despite that, there is no clear way of defining a gerrymander. This is due to its limitations in that one could not solely consider its intentions. This is due to its lack of clarity and could not possibly be known unless there is clear and direct evidence. Thus, there is no better way to examine a gerrymander through its application of quantifiable analysis that will attempt to address the issue of partisan asymmetry.

Criteria for Fairness and Unfairness

Warrington (2019) proposes two postulates that underpin the evaluation of fairness in districting:

Linear Vote Distribution: An impartial division formula is one that has a linear distribution of votes for each side. For a list sorted in ascending order, a straight line would be followed by the vote fractions, indicating that a uniform change in votes translates to a similar change in seats. This is represented by seats-votes relationship graphs that form symmetrical shapes.

Symmetry in Winning Margins: An unethical district map is one in which the average margin of victory for the majority party is substantially lower than that for the minority party. This difference stems directly from packing and cracking in as much as the minority party is required to 'waste' votes in a landslide victory while the majority party wins by a narrower margin in more districts.

These postulates, though not comprehensive, represent a set of minimal conditions under which the fairness of a district map might be properly assessed. They purposely sidestep the question of competitiveness in favor of partisan symmetry (Warrington, 2019). This is particularly important, since non-competitiveness could be a result of a variety of non-gerrymandering causes, such as the physical distribution of the voting populace.

The Role and Limits of Geography and Compactness

Traditionally, irregularly shaped legislative districts have long been regarded as a possible manifestation of a gerrymander. Measures of compactness, such as Polsby-Popper indexes and Reock indexes, provide a quantifiable assessment of regularity with respect to shape. Nonetheless, a tortuous boundary and a partisan gerrymander are not necessarily synonymous. In other words, a perfectly compact geometrical division can be a grossly unfair one if voter distribution is unbalanced through packing and packing and packing-related cracking. Again, irregular shape might be a valid shape if it satisfies Voting Rights Act and geographic considerations.

Therefore, whereas compactness measures have remained relevant as secondary indicators, the mainstream application of quantitative indicators has shifted to the distribution of votes in districts.

However, as shall be argued in this paper, it is possible for these indicators to disregard a possible gerrymander in districts where discontinuities exist.

COMPARATIVE ANALYSIS OF EXISTING QUANTITATIVE MEASURES

The Efficiency Gap

The Efficiency Gap (Ref. 3) is one of the most well-known indices used to measure the extent of gerrymandering by party, having made headlines since its appearance in the case Whitford vs. Gill. The Efficiency Gap measures the difference between the wasted votes of both parties in the election, according to whether these votes were given to the winning party or the losing party. The Efficiency Gap equation is given by:

$$EG = \frac{Total\ waste\ votes\ of\ Party\ A - Total\ waste\ votes\ of\ Party\ B}{Total\ votes\ casted}$$

A value of zero indicates perfect symmetry, while values significantly greater than zero suggest a partisan advantage for one party.

Advantages:

Intuitive interpretation as a form of vote wastage.

Sensitive to both packing and cracking strategies.

Operationally simple and widely understood.

Disadvantages:

Assumes equal turnout across districts.

Is less reliable in small legislatures or where one party overwhelmingly dominates the vote.

Can be manipulated by strategic candidate placement or unopposed races.

Does not consider underlying demographic or geographic factors, potentially conflating natural advantage with intentional gerrymandering.

Despite its popularity, however, the EG has been criticized for its inability to differentiate between deliberate and incidental asymmetry, and its susceptibility to both the number of districts and statewide percentage votes.

Declination

The Measure of Declination was proposed by Warrington in 2019 as an expression that attempts to assess how sensitive each party is to vote swings. The measure is calculated based on the average vote margins and average seat shares won by each of the parties. It is expressed geometrically as the angle created by two lines that are fitted to the ordered district vote shares.

Advantages: Robust across a wide range of hypothetical and empirical elections.

Avoids many false positives and negatives, particularly in detecting the effects of packing and cracking.

Less sensitive to the overall statewide vote share, especially in moderate-sized legislatures.

Disadvantages: Undefined in cases where one party sweeps all seats.

Interpretation can be less intuitive for non-specialists.

Like the EG, does not incorporate demographic or geographic context.

Warrington (2019) concludes that the Declination is the most successful general-purpose voting rule of the voting rules considered in preventing misclassifications of fair or unfair elections.

Partisan Bias

Partisan Bias (Ref. 4) measures the difference in seat shares each party would achieve when each party wins 50% state-wide voting. Partisan Bias is commonly defined using the uniform swing method to calculate the number of seats each party could win if it was equal.

Advantages: Directly examines symmetry on seats-votes translation.

Aids comparative analysis across plans and elections.

Disadvantages: Based on the counterfactual assumption that there is a tied vote state-wide, which is not necessarily true.

May be unstable in respect to variations of votes.

It is not very effective in identifying both the packing and cracking of products.

Mean-Median Difference

The Mean-Median Difference (MM) is a measure of the asymmetry in the distribution for district-level partisan vote shares. It is defined by the formula MM = |Mean-Median| for a particular party. (Ref. 5)

Advantages: Easy to calculate and understand.

Sensitive to skewness caused by packing or cracking.

Disadvantages: Insensitive to certain kinds of gerrymandering.

Poor performance in outlier tests; unable to point out obvious outliers.

Does not take into account the distribution of seats. It looks at the vote shares.

Comparative Performance and Limitations

Buzas & Warrington (2021) carry out detailed comparative assessments of these indices, based either on past results or simulated elections. The findings of these papers can be summarized as below:

- The Declination and Efficiency Gap are generally more sensitive and robust indices than Partisan Bias or Mean-Median Difference for gerrymandering.
- The EG and Declination function effectively when simulating packing and cracking, consistently highlighting the manipulated plans.
- Partisan Bias and MM are known to suffer from false negatives, especially when gerrymandering is effectuated through subtle changes of voters rather than through deliberately imbalanced seat allocations.
- None of the tests fully captures the demographic or geographical information necessary to identify gerrymanders based on the discontinuities of districts' boundaries.

These weaknesses cause a need for creating a new scale, combining all possible strengths from other scales while correcting for their fundamental weaknesses.

A NEW SCALE FOR MEASURING GERRYMANDERING: COMMUNITY UNIFORMITY BORDER INDEX FOR EXPOSURE (CUBIX)

Rationale and Conceptual Framework

The flaw in these measures is the lack of attention given to the cross-sectional disparities in population demographics and party affiliation. Gerrymandering, particularly the more clever variants, may rely on demographic differences that are much more subtle, resulting in a set of geographically defined voting districts that are very homogeneous in terms of demographic characteristics while being highly distinct from the neighboring areas.

The proposed Community-Uniformity Border Index for Exposure ($CUBIX$) fills this void by formally measuring demographic and partisan uniformity discrepancy separately in each pair of contiguous districts. The conceptual model is that without deliberate gerrymandering, each successive pair of districts should share similar demographic characteristics and electoral outcomes, based on natural variation. Abrupt transitions are earmarks of appreciable boundaries created to circumscribe and scatter specific voting groups.

Mathematical Formulation

Let A denote the set of all districts in a given districting plan, and $B \subseteq A \times A$ the set of ordered pairs of adjacent districts (i.e., districts that share a non-trivial border).

For each pair $(i, j) \in B$, define:

$\Delta_{dem}(i, j)$: Absolute difference in a composite demographic index (which could include race, income, age, education, etc., with appropriate weights) between districts i and j.

$\Delta_{part}(i, j)$: The absolute difference in recent (e.g., last two cycles) average partisan vote share between districts i and j.

$L\,(i, j)$: The length of the shared border between districts i and j, normalised by the perimeter of the respective districts.

The $CUBIX$ for a districting plan is then defined as:

$$CUBIX = \frac{\sum_{(i,j)} \{ L(i,j) \cdot [\, \omega_{dem} \cdot \Delta_{dem}(i,j) + \omega_{part} \cdot \Delta_{part}(i,j) \,] \}}{\sum_{(i,j)} L(i,j)}$$

where ω_{dem} and ω_{part} are weights reflecting the relative importance of demographic and partisan continuity as per the design emphasis).

Key Features

- Borders with longer shared perimeters are weighted more heavily, reflecting the greater opportunity for manipulation along extended boundaries.
- The focus on differences between adjacent districts ensures that the measure is sensitive to cross-border discontinuities, an indicator of intentional gerrymandering.
- Demography is prioritised over geography, but both are incorporated, addressing concerns that compactness alone is insufficient.
- The use of recent electoral cycles (e.g., last two) provides robustness against one-off anomalies, as recommended in best practices.

Interpretation

A low $CUBIX$ value reflect similarity in demography and politics between contiguous districts, implying a good match between voting district boundaries and natural groups, and thus avoidance of splitting and lumping voters for political purposes. High scores for *CUBIX* are conclusive indicators of gerrymandering, especially where there are discontinuities not accounted for by geography and representation mandates.

The index is meant to be a continuous scale, not a pass/fail rate. This allows for subtlety and the ability to compare rates between plans and between general jurisdictions.

Special Cases and Edge Considerations

Districts with Poor Compactness

If a district is a “thin ribbon” but its demographics and partisan outcomes are continuous with its neighbours, the $CUBIX$ will not penalise it very heavily, reflecting the design priority of demographic over geometric regularity (see Q&A). Nevertheless, the presence of multiple such districts may warrant supplementary review.

Short vs. Long Borders

Borders that are long are given more weight, since manipulation is a bigger concern on longer borders. Corner borders, and borders that are from point to point, do very little.

Rolling Averaging

Averaging over the previous two elections prior to redistricting ensures the index is quite robust to fluctuations, yet retains information about underlying patterns of partisan behavior.

EMPIRICAL STRATEGY FOR VALIDATION

Simulation-Based Evaluation

The $CUBIX$ model can also be proven correct through the use of the 'simulated packing and cracking' or SPC approach. In this process, the historical or simulated voting patterns would be varied to achieve the scenario created by the use of gerrymandering.

Key steps include:

Developing a set of sample plans for districts using varying levels and types of gerrymandering.

Calculation of $CUBIX$, EG, Declination, Partisan Bias, and MM for each plan.

Evaluation of the sensitivity as well as specificity of *CUBIX* compared with current standards for plans in which cross-border discrepancies have been introduced without obvious alterations in overall seat representation.

Historical Case Studies

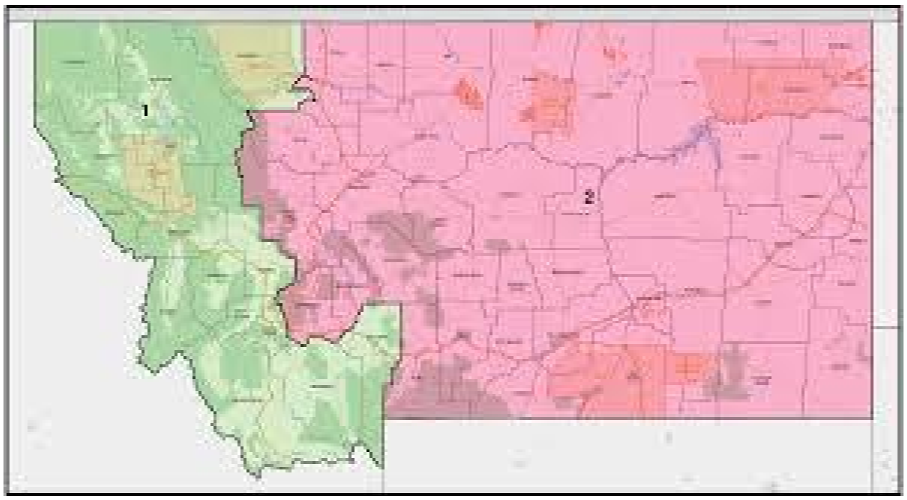
The application of the *CUBIX* to real district plans, such as that of the 2012 North Carolina US House election, enables comparisons with expert and judicial evaluations. Those plans considered to be highly gerrymandered should have higher *CUBIX* scores, while those considered fair should have low scores. Where there are discrepancies, qualitative analysis can establish whether the *CUBIX* is picking up forms of manipulation not previously identified.

Integration with Simulation Baselines

The *CUBIX* can be contextualised using computer-generated ensembles of district plans. Comparing the observed *CUBIX* for a given plan to the distribution of *CUBIX* values across simulated (non-partisan) plans permits evaluation of the extremity of the observed plan, hence the likelihood of intentional gerrymandering.

MEASURING GERRYMANDERING IN THE UNITED STATES

US Congressional Districts in Montana



The image above (Ref. 6) shows latest redistricted map of Montana based on 2020 census used for 2022 and 2024 US House of Representatives elections. We have to calculate CUBIX index for this redistricting plan.

To consider:

- Values of ω_{dem} and ω_{part} (taking both equal here, both must be 0.5)
- The measure of demography (taking racial composition between whites and people of colour)

Step 1: Calculate $\Delta_{dem}(i, j)$

As per 2020 Census, the racial composition of both districts is as follows (Ref. 7, Ref. 8)

District	White	Colour
1st	84.8%	15.2%
2nd	81.4%	18.6%

$\Delta_{dem}(i, j) = 3.4$

Step 2: Calculate $\Delta_{part}(i, j)$

As per 2024 US House of Representative Election results (Ref. 9)

District	Republican Party	Democratic Party
1st	52.3%	44.6%
2nd	65.7%	33.9%

As per 2022 US House of Representative Election results (Ref. 10)

District	Republican Party	Democratic Party
1st	49.6%	46.5%
2nd	Not to be counted	Not to be counted

*Due to entrance of a major independent candidate which split through democratic and republican votes in unknown ratio, only 2024 US House elections shall be considered for Montana's 2nd Congressional District.

Taking Average

District	Republican Party	Democratic Party
1st	50.9%	45.6%
2nd	65.7%	33.9%

$\Delta_{part}(i, j) = \frac{(65.7-50.9)+(45.6-33.9)}{2} = 13.25$

Step 3: Calculate (i,j)

Perimeter of Montana's 1st Congressional District: 2621.26 km (1628.77 miles)

Perimeter of Montana's 2nd Congressional District: 2591.21 km (1610.10 miles)

Border between the two = Sum of Perimeter of both districts – Perimeter of the state of Montana = 1628.77 + 1610.10 – 1947 = 1291.87 miles (or 2079.06 km)

$$L\,(i, j) = \frac{\textit{Border between the two districts}}{\textit{Sum of perimeter of both districts}} = \frac{1291.87}{1628.77+1610.10} \approx 0.40$$

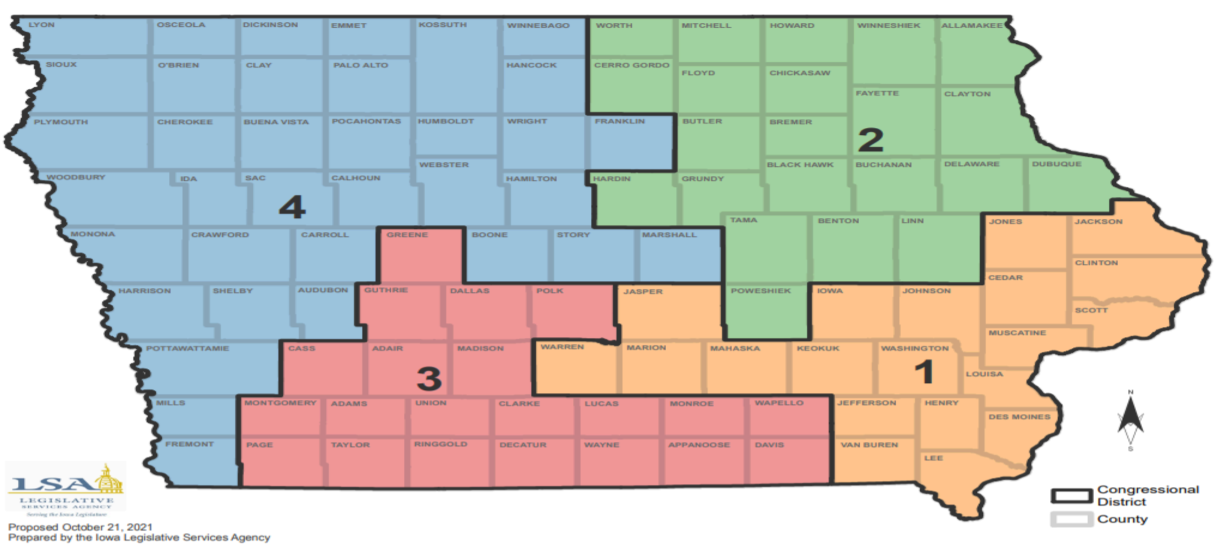
$$CUBIX = \frac{\sum_{(i,j)} \{\, L\,(i,j) \cdot [\, \omega_{dem} \cdot \Delta_{dem}(i,j) + \omega_{part} \cdot \Delta_{part}(i,j)\,]\,\}}{\sum_{(i,j)} L\,(i,j)}$$

Finally, on filling the data

$$CUBIX = \frac{0.40(0.5\times3.40+0.5\times13.25)}{0.40} \approx 8.3$$

For more than two districts, we shall use summation function and use data for all combination of districts.

US Congressional Districts in Iowa



The image above (Ref. 11) shows the latest redistricted map of Iowa based on the 2020 census used for 2022 and 2024 US House of Representatives elections. We have to calculate CUBIX index for this redistricting plan.

To consider:

- Values of ω_{dem} and ω_{part} ($\omega_{part} = 2\omega_{dem}$, then $\omega_{part} \approx 0.7$ and $\omega_{dem} \approx 0.3$)
- The measure of demography (taking racial composition between whites and people of colour)

Step 1: As there are more than 2 districts here, several combinations are to be taken, collect the data required for calculation. (Ref. 12 to Ref. 18)

Districts	Perimeter (miles)	Racial Data		Partisan Data		
		Whites	Colour	Year	Republican	Democratic
1st	696.5	83.4	16.6	2022	53.4	46.6
				2024	50	49.8
2nd	624.2	85.9	14.1	2022	54.2	45.8
				2024	57.1	41.5
3rd	619.7	78.8	21.2	2022	50.2	49.6
				2024	51.8	47.9
4th	991.5	82.6	17.4	2022	67.3	30.4
				2024	67	32.7

Step 2: Take average of partisan data

District	Political Parties	
	Republican Party	Democratic Party
1st	51.7	48.2
2nd	55.6	43.7
3rd	51.0	48.8
4th	67.1	31.6

Step 3: From the data, calculate all required quantities for combination of two districts

Combination of Districts (i and j)		$L\ (i,j)$		$\Delta_{part}(i,j)$			$\Delta_{dem}(i,j)$
		Border between them	$L\ (i,j)$	Diff. in Republican vote share	Diff. in Democratic vote share	$\Delta_{part}(i,j)$	
1st	2nd	120.3	0.1	3.9	3.5	3.7	2.5
	3rd	104.9	0.1	0.7	0.6	0.7	4.6
	4th	24.2	0.1	16.4	16.6	16.5	0.8
2nd	3rd	0.0	0.0	4.6	5.1	4.8	7.1
	4th	96.1	0.1	11.5	12.1	11.8	3.3
3rd		201.2	0.1	16.1	17.2	16.6	3.8

As $CUBIX = \dfrac{\sum_{(i,j)} \{ L(i,j) \cdot [\, \omega_{dem} \cdot \Delta_{dem}(i,j) + \omega_{part} \cdot \Delta_{part}(i,j) \,] \}}{\sum_{(i,j)} L(i,j)}$

Numerator = $\sum_{(i,j)} \{ L(i,j) \cdot [\, \omega_{dem} \cdot \Delta_{dem}(i,j) + \omega_{part} \cdot \Delta_{part}(i,j) \,] \}$

$= 0.1(0.3\times2.5 + 0.7\times3.7) + 0.1(0.3\times4.6 + 0.7\times0.7) + 0.1(0.3\times0.8 + 0.7\times16.5) + 0.0(0.3\times7.1 + 0.7\times$
= 3.9

Denominator = $\sum_{(i,j)} L(i,j) = 0.1 + 0.1 + 0.1 + 0.0 + 0.1 + 0.1 = 0.5$

$CUBIX = \frac{3.9}{0.5} = 7.8$

Note that 2nd and 3rd district do not share border and hence their demography and partisan data, even if too different, does not directly imply gerrymandering. Hence the $L(2,3) = 0$ term made entire multiplier zero and balanced the impact. The scale also ensures in such a way that an attempt to concentrate a particular partisan or demographic community in one district or to divide in multiple districts is not overvalued as if any attempt is made to have a non uniform demographic composition between two non bordering districts (like 2nd and 3rd district here), the impact will be noted through the difference in partisanship/demographics between 1st and 2nd and between 1st and 3rd district will have the same difference in demographics, and so will be between 2nd and 4th district and between 3rd and 4th district. This nature arises due to the the summation of all possible combination.

ADVANTAGES AND POTENTIAL DRAWBACKS OF THE CUBIX

Advantages

Integration of Demographic and Partisan Data: Through the incorporation of both demographic data and electoral data, the *CUBIX* includes a wider range of factors for possible manipulation.

Sensitivity to Subtle Gerrymandering: The border-conscious method is very effective at capturing subtle gerrymanders that use demographic boundaries to create sophisticated, more nuanced distortions rather than simply blatant asymmetry.

Flexibility: The weighting scheme as well as the specific variables chosen for the demographic variables can be designed according to the situation.

Robustness: Recent election cycles help to neutralize the effects of abnormal elections.

Continuous Scale: Being a non-binary variable, the *CUBIX* is capable of being interpreted properly. It can be used correctly along with other variables.

Potential Limitations

Data Requirements: To calculate the accurate values for *CUBIX*, it needs demographic and electoral data that might not necessarily be available on the sub-district level.

Complexity: It is relatively demanding in terms of computation compared to simple statistics. This could hinder use by people outside the technical field. It is easy to calculate data when a redistrictign plan is about creating lesser districts, as districts increase, the combinations between districts increase and so does the number of terms involved.

Edge Cases: Where there are real discontinuities of the population or the electoral divide because of geographical or historical reasons, high scores in $CUBIX$ may accurately depict the circumstances instead of vote manipulation.

Weighing impact: As the relative impact of demographics and partisanship not only depends upon the interpretation of a person but also depends upon the culture, freedom, history of suppressed communities in and about the region as well as non-demographic diversity, it is hard for a person to calculate the impact. Though the use of a local scale survey may help determining the relative impact of partisanship and demographics, doing so will make the calculation much less transparent than its alternative methods.

Region of application: As the data depends on all the combinations of the redistricting plan (for example, a state responsible for delimitation within the state), it does not account for any form of gerrymandering which may occur due to the drawn boundary of the applicable region (state).

It shall be noted that the method can still be used to calculate the gerrymandering occurred nationwide due to the boundaries of a state by considering each state a district and the country the region of application but the method will not give a district-wise impact of gerrymandering occured.

DISCUSSION AND IMPLICATIONS

Relationship to Existing Measures

The $CUBIX$ is not designed to replace current metrics but rather to augment them. As Warrington (2019) stresses, no measure can fully capture the multifaceted nature of gerrymandering. By focusing on cross-border discontinuities, the $CUBIX$ fills a fundamental gap in current practice, putting manipulations beyond the reach of vote-distribution or compactness measures into check.

In practice, a composite approach is advised whereby plans flagged by one or more of the measures go through deeper review, including simulation-based evaluation and qualitative assessment of intent and effect.

Legal and Policy Applications

The *CUBIX* offers several advantages for legal and policy settings:

Redistricting Commissions: The measure can guide the design of fairer district plans by identifying and minimising artificial discontinuities.

Litigation: As part of a suite of evidence, the *CUBIX* can strengthen claims of intentional gerrymandering, especially where plan sponsors deny manipulative intent.

Public Engagement: Transparent reporting of *CUBIX* scores can inform public debate and increase accountability in the redistricting process.

Future Extensions

Areas for further research and refinement include:

Automated Weight Selection: Developing data-driven methods for selecting optimal weights for demographic and partisan continuity.

Integration with Compactness Measures: Exploring hybrid indices that combine *CUBIX* with geometric metrics, providing a more holistic assessment.

Dynamic Updating: Adapting the measure to reflect changing demographic and political realities over time.

International Application: Assessing the utility of *CUBIX* in non-U.S. contexts, where electoral systems and data availability may differ.

CONCLUSION

The problem of quantifying partisan gerrymanders is a rapidly changing and challenging problem that is at the junction of mathematics, statistics, and politics. While methods such as Efficiency Gap, Declination, Partisan Bias, and Mean-Median Distance have made contributions to this problem, there are certain natural flaws in each that make it difficult to point out manipulation that hinges on demographic and partisan discontinuity at boundaries.

This paper proposes Community-Uniformity Border Index for Exposure (*CUBIX*), which is a new measure that formalizes differences in both sides of the border with regard to demographic and partisan composition, with weights that are based on border lengths. The *CUBIX* measure considers continuity in each adjoining district because it fills an important gap in border measures in current practice that this paper identifies.

By theoretical and simulated validation, the *CUBIX* appears to have utility as either a diagnostic aid and/or prevention method. Despite certain challenges associated with data collection and interpretability, the combination and intersection of demographic, partisan, and geographic data constitutes a substantial step forward in the perpetuation of fair representation.

In the end, the battle against partisan gerrymandering must be conducted using a comprehensive arsenal of methods. The *CUBIX*, as will be discussed in this paper, provides a vital tool for this fight—that of achieving a balance between mathematical expressiveness and real-world usability.

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