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# Mathematical Model for Accessibility of Extraterrestrial Civilizations

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## ABSTRACT

A lot of work has been done on the number and chance of detecting extraterrestrial life, but humanity's accessibility to communication with them, such as a compatible time window or compatible means of communication, has been largely underrated. This investigation analyzes the accessibility of extraterrestrial civilizations to communicate with human civilization and, more importantly, the chance of a galaxy containing accessible life. A galaxy or system containing civilizations is not considered useful unless they are accessible. This paper evaluates certain factors that make a civilization accessible and combines them with the Drake equation to yield an extended equation that details the chance of detecting accessible extraterrestrial habitats within a galaxy. However, this model can very easily be altered to fit most units of habitat(such as a stellar system)

## 1 INTRODUCTION

The scientific community has established several projects in the search for extraterrestrial life, with the prominent ones being the Search for Extraterrestrial Intelligence(SETI) and the Messaging Extraterrestrial Intelligence(METI). These projects use models that estimate the chance of extraterrestrial intelligence being present in certain areas that have been substantially developed. Cocconi and Morrison in 1959, and Frank Drake in 1961, have developed models and equations, such as the Drake Equation, to estimate the number of active, communicable civilizations and the probabilities of their existence. Additionally, these models have been expanded and integrated with the probability of life arising on habitable planets (Cartin, 2015), James Miller's Living Systems Theory (Harrison, 1993), etc. However, an important consideration has been missed: accessibility. Many systems might hold active, communicable life, but, whether because of their lack of a habitable time window (Rossmo, 2017), or their lack of a shared means of communication, are not accessible to human civilization specifically. Even though it might have a means of communication, it might not be compatible with that of Humanity. A lack of a model for accessibility can serve a strain on search projects due to the limited funding and size of programs for detecting or sending signals to civilizations (Shostak, 2015). Thus, such a model must be used to evaluated which civilizations can actually be accessed, as a focus on such civilizations would create responsible use of funding. In light of the issue, this paper sets out to develop a model for (i) an accessibility score that states how accessible a civilization is to humans and (ii) an implementation of the accessibility score with the Drake Equation to develop a model of the chance of detecting extraterrestrial intelligence that is actually accessible for human civilization to detect.

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## 2 FACTORS OF ACCESSIBILITY AND DERIVATIONS

This paper will approach the problem by considering each factor that plays a role in the accessibility of communication with civilizations. Then, each factor will be combined into equations. From here, the equations will be used in one equation for an accessibility score, a rating of how accessible a civilization is, assuming its existence. Finally, the assumption will be removed, using the accessibility score to develop a model for the chance of detecting accessible civilizations in a galaxy.

### 2.1 Identifying factors

First, the variables that contribute to the accessibility of civilizations will be identified.

**Time Window to Emit Signals**: There needs to be a certain window of time in which humans can send a signal such that it is received while the civilization is still inhabited and communicable, while considering the time for the signal to reach (Rossmo, 2017).

**Change in Emitted Frequencies**: The frequencies that will be received will be different than that sent due to interference and the Doppler Effect.

**Receiver's Access to Frequencies**: The receiver must have the ability to access and read the frequencies they receive from human signals.

**Number of Extraterrestrial Civilizations**: The actual number of civilizations available before accessibility is even accounted for needs to be accounted for.

### 2.2 Time Window to Emit Signals

There are two considerations when solving for a window of time in which signals can be sent. First, the amount of time it will take to reach the galaxy is considered. Second, this paper shall consider the existence of the civilization when the signal reaches. The amount of time to reach will be considered first.

Edwin Hubble proved in 1929 that the distance between galaxies and the Milky Way increases over time (Hubble, 1929). To account for this, this paper will let $a$ be the scale factor of space, $\dot{a}$ be the derivative of the scale factor with respect to time, $R$ be the coordinate distance between two galaxies, $t_{sent}$ as the time the signal was sent, $\Delta t$ as the amount of time taken to reach another planet, and $c$ as the speed of light.

The distance between the Milky Way and the receiving galaxy will be $aR$, to account for the physical distance instead of simply the coordinate one. The velocity at which the galaxy will be traveling away will be $\dot{a}R$. This velocity will need to be converted into a distance in order to find the extra distance that the galaxy travels while the signal is traveling, which can be done by multiplying by time. Thus, the increase in distance at the time the signal is received from the time the signal was sent will be $\dot{a}R\Delta t$. One can now add this to the original distance to derive,

Since the signal is an electromagnetic wave, the velocity will be the speed of light, $c$. As a result, the distance the signal must travel will be $c\Delta t$. The distance the signal travels must, for obvious reasons, equal the distance between the Milky Way and the receiving galaxy at the time it is received. Thus, the two expressions can be equated in an effort to solve for $\Delta t$.

$c\Delta t = aR_{sent} + \dot{a}R\Delta t$ (2)

$c\Delta t - \dot{a}R\Delta t = aR_{sent}$ (3)

$(c - \dot{a}R)\Delta t = aR_{sent}$ (4)

$$\boxed{\Delta t = \frac{aR_{sent}}{c - \dot{a}R}} \quad (5)$$

This paper also considers the existence of the civilization when the signal reaches.
Let $L_S$ be the time a civilization begins its ability to communicate, and $L_E$ be the time
a civilization has ended its ability to communicate, whether it is due to a collapse or other reasons. For obvious reasons, it can be written:

$L_S \leq t_{sent} + \Delta t < L_E$ (6)

$$\boxed{L_S \leq t_{sent} + \frac{aR_{sent}}{c - \dot{a}} < L_E} \quad (7)$$

**2.3 Change in Emitted Frequencies**

This investigation will also consider frequencies emitted. Specifically, the frequencies that will be received will be different than the frequencies sent, due to interference and the Doppler Effect. First, the Doppler Effect will be considered. The Relativistic Doppler Effect, which is used because electromagnetic waves are the object of analysis, is written as

$$f = f_0\sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}} \quad (8)$$

This analysis will measure time and distance from humanity's frame of reference. The Milky Way, from the Milky Way's frame of reference, is constant, while the target galaxy is moving away at a velocity of $\dot{a}R$. Also, the velocity of an electromagnetic signal through empty space is $c$. Thus, it is derived:

$$f = f_0\sqrt{\frac{1 - \frac{\dot{a}R}{c}}{1 + \frac{\dot{a}R}{c}}} \quad (9)$$

$$f = f_0\sqrt{\frac{c - \dot{a}R}{c + \dot{a}R}} \quad (10)$$

For the interference of the signal, as well as the frequencies that can be received, which will be dealt with later, great inspiration comes from "Searching for Interstellar Communications" by Giuseppe Cocconi and Philip Morrison. In it, it is discussed that the signal "must compete with...the emission of its own local star...the galactic emission," (Cocconi & Morrison, 1959). This shall be generalized to all celestial bodies that emit a frequency. In order to properly model the frequency that humanity must be able to send, such that, after competing with celestial emissions, it is receivable by the target civilization, it is important to include this in the model. Thus, the frequencies of the emissions from celestial bodies shall be written as $S_n$, where $n$ is the number to enumerate the bodies and $A(S)$ is the number of celestial bodies. The interference from external emissions can be written via the mathematical procedure of accounting for constructive and destructive interference, as

$$\sum_{n}^{A(S)} S_n \quad (11)$$

This interference shall now be added to the original Doppler Effect results to yield:

$$f_F = f_0\sqrt{\frac{c - \dot{a}R}{c + \dot{a}R}} + \sum_{n}^{A(S)} S_n \quad (12)$$

where $f_F$ is the final frequency that will be received.

### 2.4 Receiver's Access to Frequencies

Another important factor to consider is the frequencies the target has access to. Different civilizations have different access to certain frequencies, so it is important to identify which frequencies are accessible. This is especially important because improvements in radio technology have "broadened the range of frequencies" that can be sent or detected (Shostak, 2015). This paper will approach the problem by identifying an ideal frequency, which the civilization has the most access to, and building a range of accessible frequencies from there, with accessibility decreasing as the frequency gets farther from the ideal.

"Searching for Interstellar Communications" uses the frequency of Hydrogen's spectral emission to label as a frequency to which most civilizations have access to because hydrogen is stated as a resource that can be found almost anywhere (Cocconi & Morrison, 1959). However, while it can work as a benchmark in general, it can work better to identify the best frequency for each galaxy specifically, catering to each target. The same rationale that was used in the paper can be used for every specific galaxy: by using the chemical composition of a galaxy to find which frequencies they have access to.

More specifically, each element has a certain quantity of the chemical composition in a galaxy. This can be multiplied by its corresponding frequency of the atoms' spectral emission, to yield an ideal frequency.

Let ***W*** be the vector of the percentage of each element in the chemical composition(i.e., [0.7 {For Hydrogen}, 0.2 {For Helium},...]). Let ***fS*** be the vector of all respective frequencies(i.e., 1420 MHz, $725 * 10^6$ MHz,...). From the prior rationale, the following can be deduced:

$f_{ideal} = \boldsymbol{W} \cdot \boldsymbol{fS}$ (13)

### 2.5 General Statement of Frequencies

Using the results from 2.2 and 2.3, a general range of frequencies can be found. Since the frequencies that the receiver must receive are different from the frequencies sent, the frequency received must match the frequencies accessible, most ideally, matching the ideal frequency. Thus, $f_F = f_{ideal}$, most ideally. Using Equation 2 and the equation from 2.3, a general equation stating the frequencies that can be sent can be yielded as:

$$\mathbf{W} \cdot \mathbf{fS} = f_0\sqrt{\frac{c - \dot{a}R}{c + \dot{a}R}} + \sum_{n}^{A(S)} Sn \quad (14)$$

However, this deals with the assumption that humanity must match the ideal frequency. One can also derive an equation that contains all accessible frequencies. Let $E$ be any frequency such that $E \leq \Xi$, where $\Xi$ is the maximum deviation from the ideal frequency that a civilization can still access. This shall yield:

$$\mathbf{W} \cdot \mathbf{fS} \pm E = f_0\sqrt{\frac{c - \dot{a}R}{c + \dot{a}R}} + \sum_{n}^{A(S)} Sn | E \leq \Xi$$

where E serves to be the deviation of the frequency received from the ideal frequency.

### 2.6 Number of Extraterrestrial Civilizations

In 1961, Frank Drake proposed the Drake Equation, an equation detailing the number of active civilizations that can be communicated with (Drake, 1965). The dependent variable of this shall be written as $N$. This will later be used in the final equations stating an accessibility constant.

## 3 SYNTHESIS OF FACTORS AND IMPLEMENTATION OF THE DRAKE EQUATION

As a result of the previous arguments, each factor has been presented. As mentioned earlier, one of the goals of the paper is to establish a formula for an accessibility score: a number that represents how accessible it is to communicate and search a civilization. This can be done by analyzing each of the listed equations and calculating the probability that each equation can be satisfied. More specifically, a function can be established that can determine the number of solutions for the desired variable. Let this function be named $\upsilon$, where $\upsilon(x)$ is the number of solutions for $x$, and $|\upsilon(x)|$ is the size of the range of solutions for $x$. Let $\upsilon(x)$ be the vector that contains every solution of $x$. Finally, let $\mathbf{\Omega}$ be the accessibility score.

This section shall begin with the frequency. One can simply draw a range of solutions out of equation 4; however, it is important to note that accessibility decreases as one draws further from the ideal frequency. It is also important to note that some civilizations will have a different impact on access of a certain deviation. For this, define $\kappa$ to be a constant, dependent on the civilization, that shall represent the decrease in access for every one unit of frequency, with the unit being the same as the one used for all frequency variables. Thus, one can state $\mathbf{\Omega}_{f\,requency} = \upsilon(f_0) \cdot \mathbf{[1 - \kappa E]}$, where $\mathbf{[1 - \kappa E]}$ is a vector of all the corresponding solutions of the stated expression $1 - \kappa E$. This dot product multiplies each possible frequency that can be sent by its corresponding ability to be received by the target civilization.

On top of this, the window of time shall be accounted for. A simple range of solutions can be drawn from Equation 1. This yields $\iota_t = |\upsilon(t_{sent})|$. These results can be synthesized into the following accessibility equation with respect to equations 1 and 4.

$$\boxed{\Omega_A = \boldsymbol{\upsilon(f_0)} \cdot \mathbf{[1 - \kappa E]} * |\upsilon(t_{sent})|} \quad (16)$$

As mentioned before, this paper also aims to develop a system to include accessibility in the Drake Equation and to deduce the chance of finding an accessible extraterrestrial habitat in a galaxy. This can be done easily by multiplying it by $N$, the number of active extraterrestrial civilizations, and the dependent variable from the Drake Equation, to finally yield, with respect to equations 1, 4, and the Drake Equation:

$$\boxed{\Omega_N = \Omega_A * N = \boldsymbol{\upsilon(f_0)} \cdot \mathbf{[1 - \kappa E]} * |\upsilon(t_{sent})| * N} \quad (17)$$

$\mathbf{\Omega}_A$ is defined as the accessibility score of a galaxy. $\mathbf{\Omega}_N$ represents the number of accessible civilizations in a galaxy.

## 4 INTERPRETATION

This investigation has now established models to measure accessibility and the chance of accessible civilizations existing and will now proceed to interpret the models into concrete results.

In Equation 16, $\kappa E$ is subtracted from 1. This reasonably implies that the value of $\kappa$ has a negative correlation with the accessibility of a civilization, which further implies that the impact of one hertz being lower increases the access of a civilization to human signals.

In Equation 15, $E$ is a variable that represents the received frequency's deviation from the ideal frequency. It is bounded by $\Xi$. It can be observed that a larger value of $\Xi$, or a larger range of decodable signals, is positively correlated with $|\upsilon(f_0)|$, which is positively correlated with the accessibility score.

In Equation 7, it is apparent that a larger $L_E - L_S$, or a larger amount of time that a civilization exists, will increase the time window to send signals(increase $|\upsilon(t_{sent})|$), which will increase the accessibility score.

Finally, in Equation 17, it is obvious that an increased accessibility score of a galaxy is positively correlated with an increased chance of finding an accessible intelligent extraterrestrial civilization in a galaxy.

## 5 CONCLUSION AND OPEN PROBLEMS

This investigation has been concluded by establishing a model for a rating of the accessibility of a civilization or other unit of space that can contain life (Equation 16), a model for the chance of finding accessible extraterrestrial intelligence (Equation 17), and concrete implications of these models (Section 4). Along with this, equations regarding frequencies that target civilizations will have access to (Equations 14 and 15) and equations regarding the time window that signals can be sent in (Equation 7) have also been established.

Due to possible uncertain variables, such as $\kappa$ or N, this paper recommends the usage of comparison of $\Omega$ values to decide where the possibility of accessible extraterrestrial life is. This model measures the key aspects that make civilizations accessible, so a comparative approach would yield that the highest value for $\Omega$ would correspond to the most likely area with extraterrestrial intelligence accessible to humans, while also accommodating for the variables that are less certain due to the limited present knowledge on the nature of extraterrestrial life.

However, there still exist open problems, namely methods to track certain variables. The $\kappa$ variable and the $\Xi$ variable do not seem to have a simple method to determine. This paper also approaches the problem at the galactic level. While it can mostly be applied to any level, there are some levels that require additional variables, namely planets or regions within galaxies (Grimaldi, 2017), which would benefit from taking habitability (Ulmschneider, 2003) and the Goldilocks zone into account. Finally, these models are beneficial for projects sending signals into space to detect extraterrestrial life. However, the question of how to extend the model to benefit projects that simply try to detect signals from extraterrestrial intelligence remains.

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